Formal Languages and Automata Theory
Exercise sheet 2: Push-down Automata

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Definition 1 *A push-down automaton (PDA) M is a tuple $\langle Q, \Sigma, \Gamma, \delta, q_0, Z_0, F\rangle$ with*

- *Q is a finite set of states.*
- *Σ is a finite set, the input alphabet.*
- *Γ is a finite set, the stack alphabet.*
- *$q_0 \in Q$ is the initial state.*
- *$Z_0 \in \Gamma$ is the initial stack symbol.*
- *$F \subseteq Q$ is the set of final states.*
- *$\delta : Q \times (\Sigma \cup \{\epsilon\}) \times \Gamma \to \mathcal{P}_{fin}(Q \times \Gamma^*)$ is the transition function. ($\mathcal{P}_{fin}(X)$ is the set of finite subsets of X).*

Definition 2 *An instantaneous description of a PDA is a triple (q, w, γ) with*

- *$q \in Q$ is the current state of the automaton,*
- *$w \in \Sigma^*$ is the remaining part of the input string, and*
- *$\gamma \in \Gamma^*$ is the current stack.*

There are two alternatives for the definition of the language accepted by a PDA $M = \langle Q, \Sigma, \Gamma, \delta, q_0, Z_0, F\rangle$:

Definition 3 *The language accepted by M with a final state is*

$L(M) := \{w \mid (q_0, w, Z_0) \overset{*}{\vdash} (q_f, \epsilon, \gamma)$ *for a* $q_f \in F$ *and a* $\gamma \in \Gamma^*\}$

Definition 4 *The language accepted by M with an empty stack is*

$N(M) := \{w \mid (q_0, w, Z_0) \overset{*}{\vdash} (q, \epsilon, \epsilon)$ *for a* $q \in Q\}$

Exercise 1 *Prove that the two modes of acceptance are equivalent, i.e., for each language L: there is a PDA M_1 with $L = L(M_1)$ iff there is a PDA M_2 with $L = N(M_2)$.*

Definition 5 *A PDA that has at most one choice of move in any state is called a deterministic PDA. Non-deterministic PDA (NPDA) provides non-determinism in the moves defined. A class of languages that are generated by PDAs is called DCFL (deterministic context free languages).*

Exercise 2 *Construct a PDA that accepts the language of palindromes. Is it a NPDA or a DPDA?*

Exercise 3 *Explore whether DCFLs are closed under complementation.*

Exercise 4 *Prove the equivalence of CFGs and PDAs.*