Linear Coding of non-linear Hierarchies: Revitalization of an Ancient Classification Method

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The Problem:

Sometimes we are forced to order things (nearly) linearly, e. g. in . . .

Libraries

Warehouses

Stores

The problem is old!

But more than 2 000 years ago it has been solved!

Pāṇini's Grammar of Sanskrit (ca. 350 BC)

- Sanskrit: rich morphology, complex Sandhi-system
- linguistics in ancient India:
  - *śāstrānāṃ śāstram* 'science of sciences'
  - oral tradition
  - Pāṇini's grammar: system of more than 4. 000 concise rules
  - many phonological rules

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Phonological Rules

modern notation

A is replaced by B if preceded by C and succeeded by D.

$$A \rightarrow B / _{C\_\_D}$$

example: final devoicing in German

$$\begin{bmatrix} + & \text{consonantal} \\ - & \text{nasal} \\ + & \text{voiced} \end{bmatrix} \rightarrow \begin{bmatrix} + & \text{consonantal} \\ - & \text{nasal} \\ - & \text{voiced} \end{bmatrix} / \_\_\sharp$$

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Pāṇini's linear Coding

$$A + \text{genitive}, B + \text{nominative}, C + \text{ablative}, D + \text{locative}.$$

example

- *sūtra* 6.1.77: *iko yaṇaci* (इको यणचि)
- analysis: $[\text{ik}]_{\text{gen}}[\text{yaṇ}]_{\text{nom}}[\text{ac}]_{\text{loc}}$
- modern notation: [iK] → [yṆ]/__ [aC]

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Śivasūtras

1.	a	i	u			Ṇ
2.				ṛ	ḷ	K
3.		e	o			Ṅ
4.		ai	au			C
5.	h	y	v	r		Ṭ
6.					l	Ṇ
7.	ñ	m	ṅ	ṇ	n	M
8.	jh	bh				Ñ
9.			gh	ḍh	dh	Ṣ
10.	j	b	g	ḍ	d	Ś
11.	kh	ph	ch	ṭh	th	
			c	ṭ	t	V
12.	k	p				Y
13.		ś	ṣ	s		R
14.	h					L

अइउण्। ऋऌक्।
a·i·uṇ| ṛ·ḷk|

एओङ्। ऐऔच्।
e·oṅ| ai·auc|

हयवरट्। लण्।
hayavaraṭ| laṇ|

ञमङणनम्। झभञ्।
ñamaṅaṇanam| jhabhañ|

घढधष्। जबगडदश्।
ghaḍhadhaṣ| jabagaḍadaś|

खफछठथचटतव्।
khaphachaṭhathacaṭatav|

कपय्। शषसर्। हल्।
kapay| śaṣasar| hal|

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4.		ai	au			C
5.	h	y	v	r		Ṭ

- a *pratyāhāra* is a pair of a sound and a marker
- it denotes the continuous sequence of sounds in the interval between the sound and the marker

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iK= ⟨i, u, ṛ, ḷ⟩

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Analysis of iko yaṇaci: [iK] → [yṆ]/_ [aC]

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5.	h	y	v	r		Ṭ
6.					l	Ṇ

- [iK] → [yṆ]/_ [aC]
- ⟨i, u, ṛ, ḷ⟩ → ⟨y, v, r, l⟩/_ ⟨a, i, u, ṛ, ḷ, e, o, ai, au⟩

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Generalized task

task

- Given a set of classes, order the elements of the classes linearly such that each class forms an interval.
- If unavoidable, duplicate some elements, but minimize the number of duplications.

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Terminology: S-sortability

set of classes $(\mathcal{A}, \Phi)$: $\mathcal{A} = \{a, b, c, d, e, f, g, h, i\}\}$

$\Phi = \{\{d, e\}, \{a, b\}, \{b, c, d, f, g, h, i\}, \{f, i\}, \{c, d, e, f, g, h, i\}, \{g, h\}\}$

S-order $(\mathcal{A} <)$ of $(\mathcal{A}, \Phi)$: $a\ b\ c\ g\ h\ f\ i\ d\ e$

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S-order $(\mathcal{A} <)$ of $(\mathcal{A}, \Phi)$: $a\ b\ c\ g\ h\ f\ i\ d\ e$

$\Rightarrow$ $(\mathcal{A}, \Phi)$ is S-sortable without duplications

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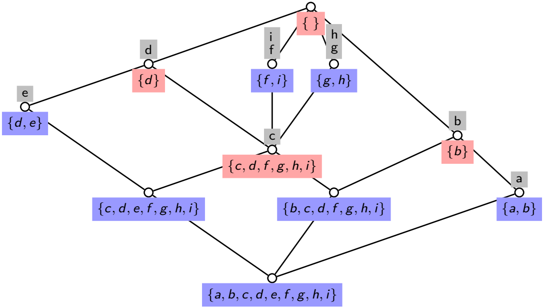

concept lattice of $(\mathcal{A}, \Phi)$

	a	b	c	d	e	f	g	h	i
$\{d, e\}$				×	×				
$\{b, c, d, f, g, h, i\}$		×	×	×	×	×	×	×	×
$\{a, b\}$	×	×							
$\{f, i\}$						×			×
$\{c, d, e, f, g, h, i\}$			×	×	×	×	×	×	×
$\{g, h\}$							×	×	

formal context of $(\mathcal{A}, \Phi)$

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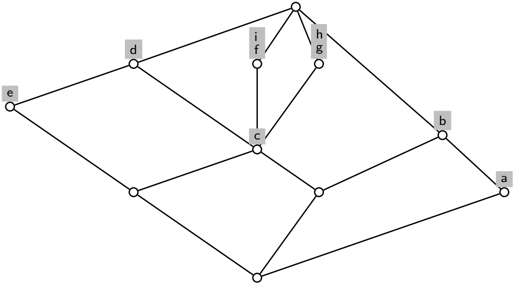

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$\{a, b\}$	×	×							
$\{f, i\}$						×			×
$\{c, d, e, f, g, h, i\}$			×	×	×	×	×	×	×
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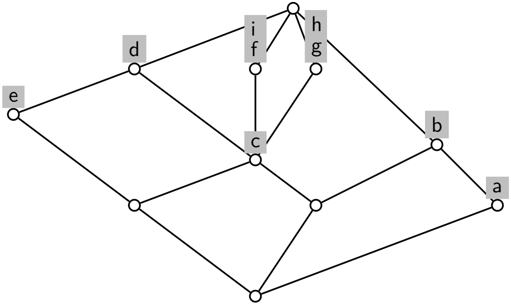
formal context of $(\mathcal{A}, \Phi)$

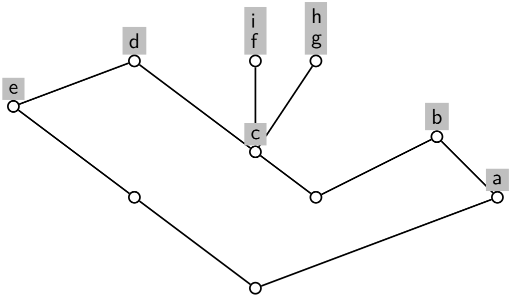
Determining S-orders



Theorem

A set of classes $(\mathcal{A}, \Phi)$ is S-sortable without duplications iff its concept lattice is a planar graph and for any $a \in \mathcal{A}$ there is a node labeled a in the S-graph.

Determining S-orders



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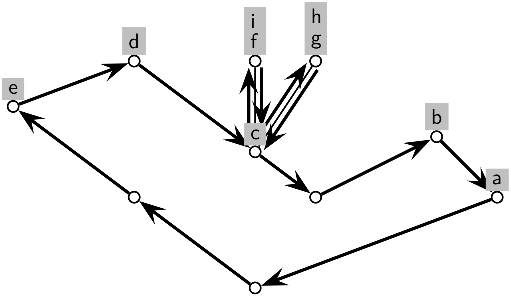

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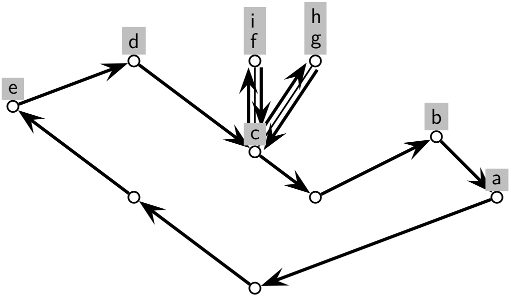

e d c f i g h b a

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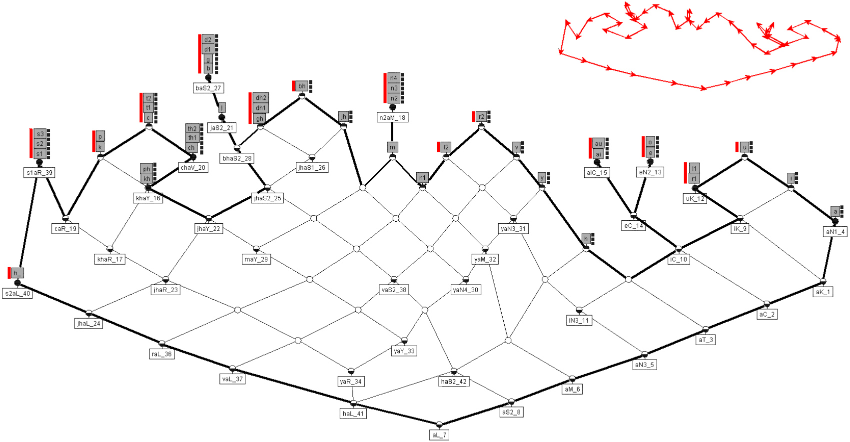
Enlarged *pratyāhāra*-concept-lattice



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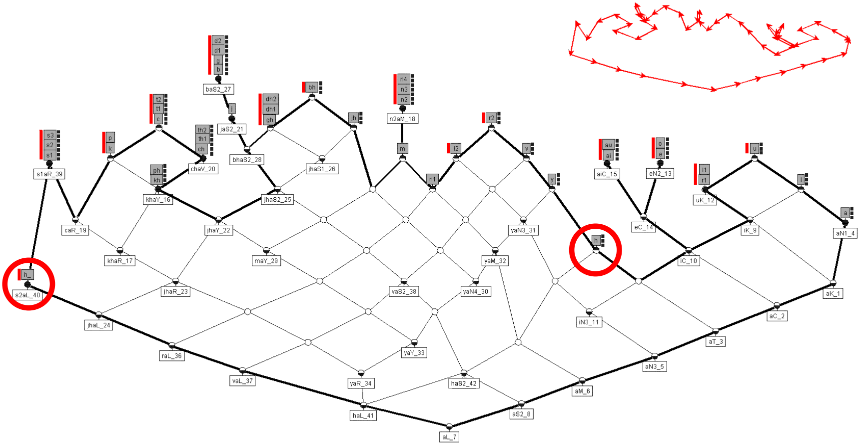


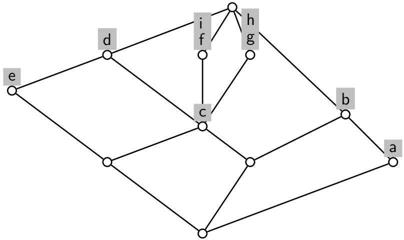
By duplicating elements all sets of classes become S-sortable!

Main theorem of S-sortability

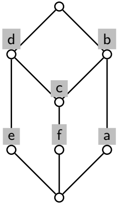
A set of classes $(\mathcal{A}, \Phi)$ is S-sortable without duplications if one of the following equivalent statements is true:

1. The concept lattice of $(\mathcal{A}, \Phi)$ is a Hasse-planar graph and for any $a \in \mathcal{A}$ there is a node labeled a in the S-graph.
2. The concept lattice of the enlarged set of classes $(\mathcal{A}, \tilde{\Phi})$ is Hasse-planar. ($\tilde{\Phi} = \Phi \cup \{\{a\} \mid a \in \mathcal{A}\}$)
3. The Ferrers-graph of the enlarged $(\mathcal{A}, \tilde{\Phi})$-context is bipartite.

Example: S-sortable



Example: not S-sortable



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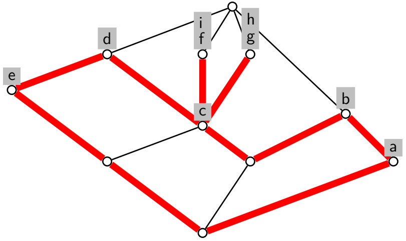

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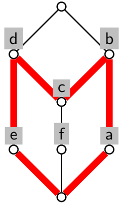

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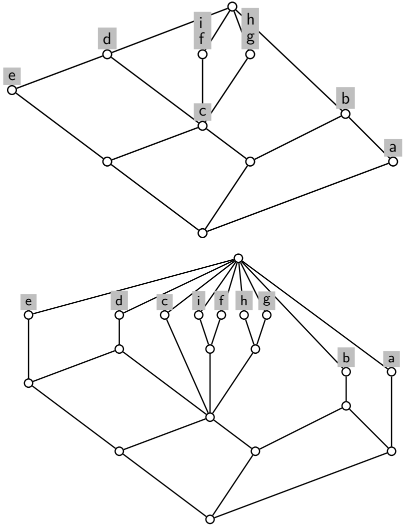

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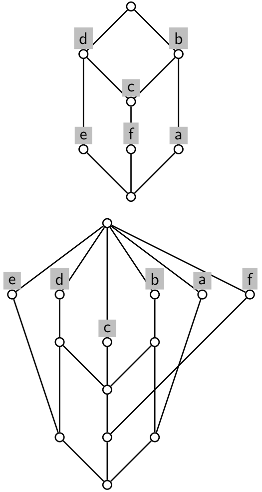

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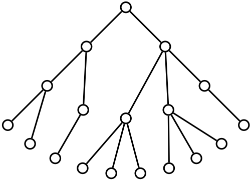
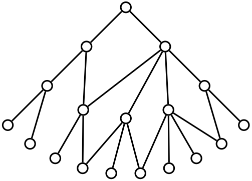
Advantages:

- The Ferrers-graph is constructed on the formal context.
- Its bipartity can be checked algorithmically.

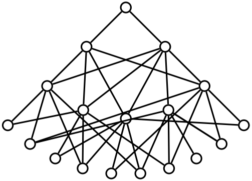
Between trees and general hierarchies



tree



S-sortable



general hierarchy

Transfer

- For physical objects ‚duplicating' means ‚adding copies'
- Adding copies is annoying but often not impossible
- Ordering objects in an S-order may
  - improve user-friendliness
  - save time
  - save space
  - simplify visual representations of classifications

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Transfer

- For physical objects ‚duplicating' means ‚adding copies'
- Adding copies is annoying but often not impossible
- Ordering objects in an S-order may
  - improve user-friendliness
  - save time
  - save space
  - simplify visual representations of classifications

Outlook

Objects in libraries, ware-houses, and stores are only *nearly* linearly arranged:

$\Rightarrow$ Second (and third) dimension can be used in order to avoid duplications

Literature

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Origin of Pictures

- libraries (left): http://www.meduniwien.ac.at/medizinischepsychologie/bibliothek.htm
- libraries (middle): http://www.math-nat.de/aktuelles/allgemein.htm
- libraries (right): http://www.geschichte.mpg.de/deutsch/bibliothek.html
- warehouses: http://www.metrogroup.de/servlet/PB/menu/1114920_l1/index.html
- stores: http://www.einkaufsparadies-schmidt.de/01bilder01/