A set-theoretical investigation of Pāṇini's Śivasūtras

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Pāṇini's Śivasūtras

sūtras

						anubandha
1.	a	i	u			Ṇ
2.				ṛ	ḷ	K
3.		e	o			Ṅ
4.		ai	au			C
5.	h	y	v	r		Ṭ
6.					l	Ṇ
7.	ñ	m	ṅ	ṇ	n	M
8.	jh	bh				Ñ
9.			gh	ḍh	dh	Ṣ
10.	j	b	g	ḍ	d	Ś
11.	kh	ph	ch	ṭh	th	
			c	ṭ	t	V
12.	k	p				Y
13.		ś	ṣ	s		R
14.	h					L

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Phonological classes/ *pratyāhāras*

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Phonological classes are denoted by *pratyāhāras.*
E.g., the *pratyāhāra* *iC* denotes the set of segments in the continuous sequence starting with *i* and ending with *au*, the last element before the *anubandha* *C.*

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Questions around Pāṇini's Śivasūtras

- Are Pāṇini's Śivasūtras perfect?
  - Is the number of stop markers minimal?
  - Is the duplication of 'h' necessary?
- Is it possible to decide whether a set of sets has a Śivasūtras-style representation?
- How to construct an optimal Śivasūtras-style representation?

Basic concepts

S-encodable set of sets: $\Phi=\{\{d,e\},\{b,c,d,f,g,h,i\},\{a,b\},\{f,i\},\{c,d,e,f,g,h,i\},\{g,h\}\}$

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S-alphabet $(\mathcal{A},\Sigma,<)$ of Φ: e d M_1 c i f M_2 g h M_3 b M_4 a M_5

alphabet

marker

total order on $\mathcal{A}\cup\Sigma$

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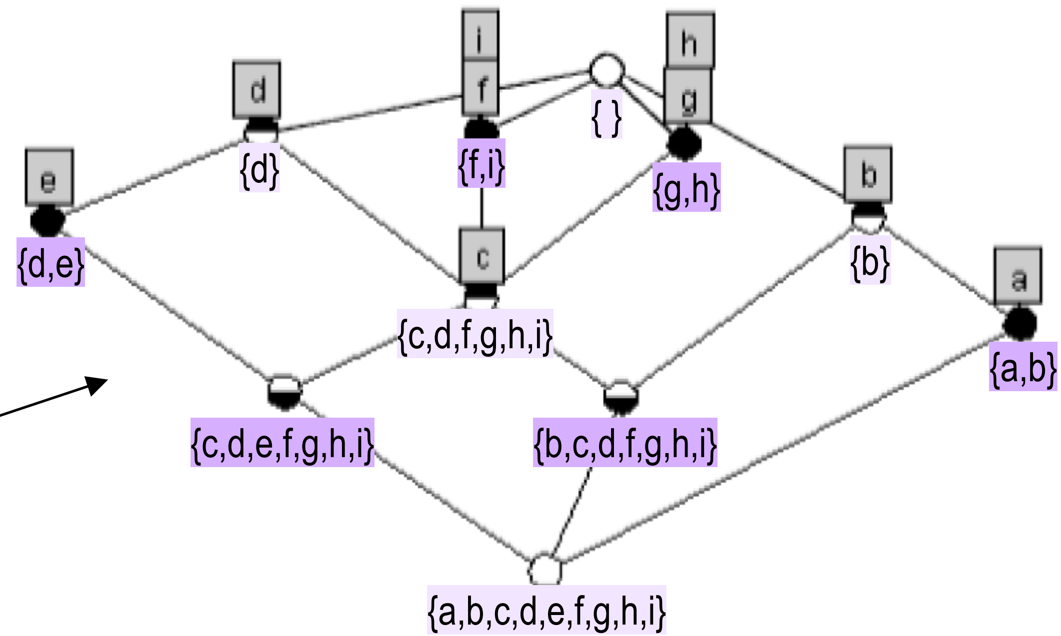
Hasse-diagram $(\mathcal{H}(\Phi),\supseteq)$ for the set of intersections of elements from $\Phi\cup\{\mathcal{A}\}$:



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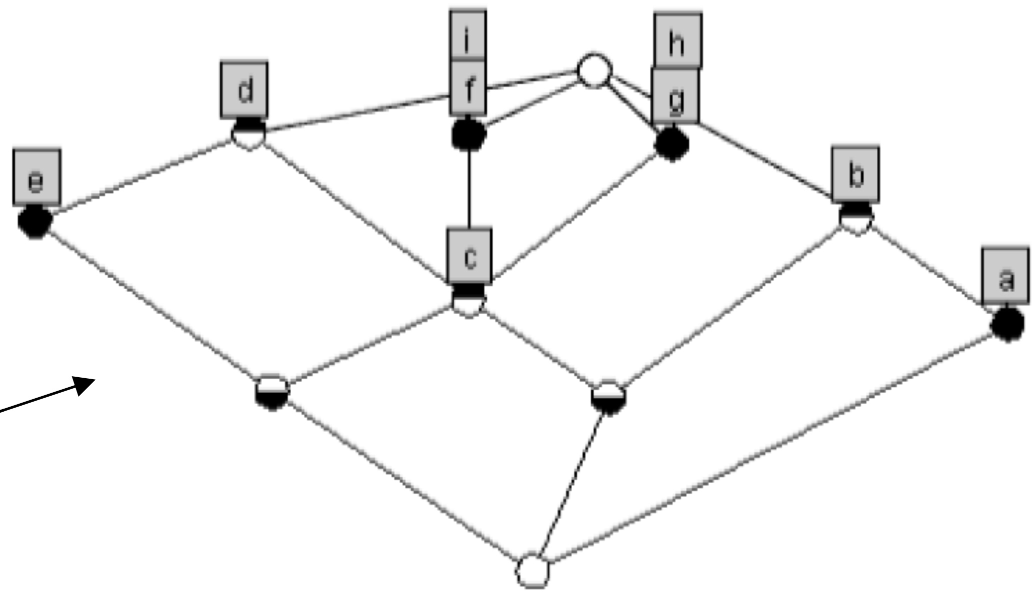
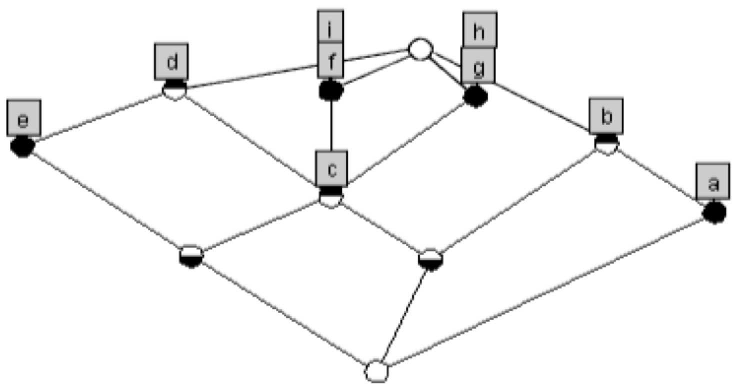


S-encodability and planar Hasse-diagrams

If Φ is S-encodable, then the Hasse-diagram of ($\mathcal{H}$(Φ),⊇) is planar



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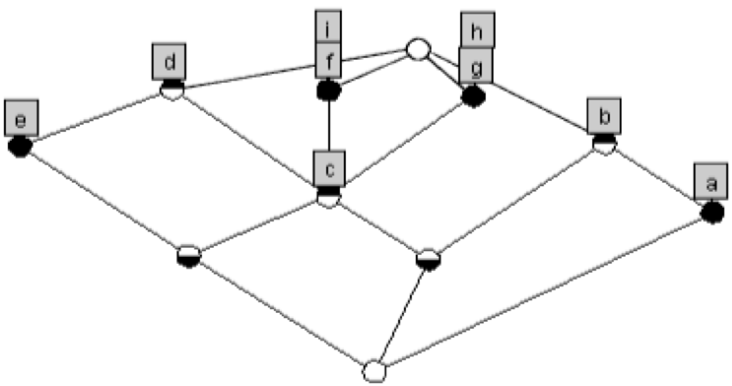
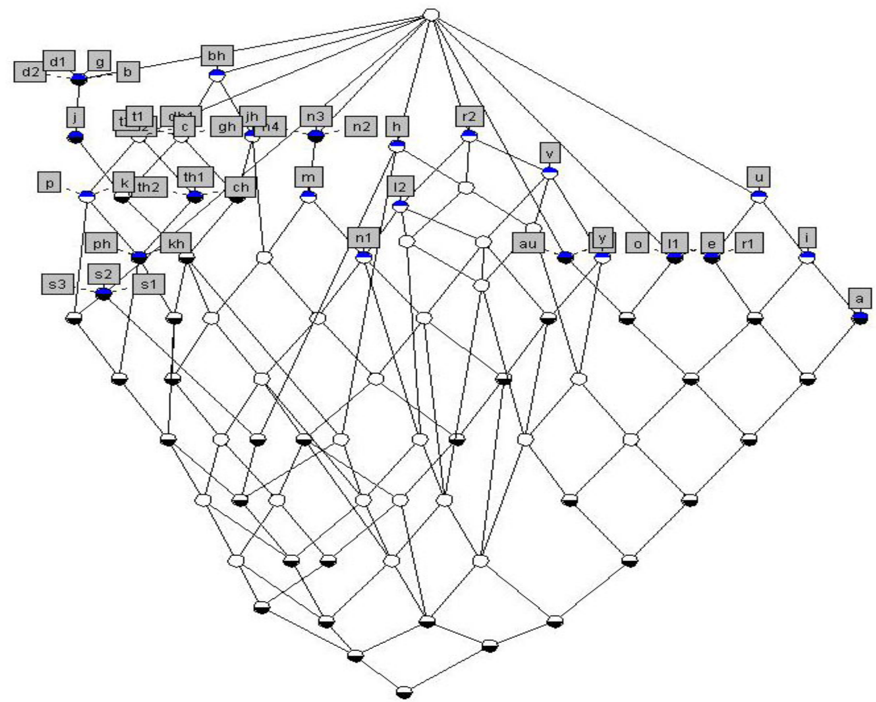
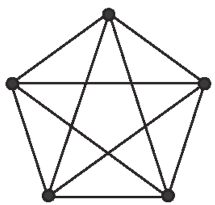




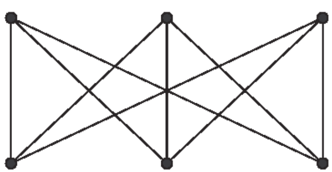
Hasse-diagram for Pāṇini's *pratyāhāras*

S-encodability and planar Hasse-diagrams

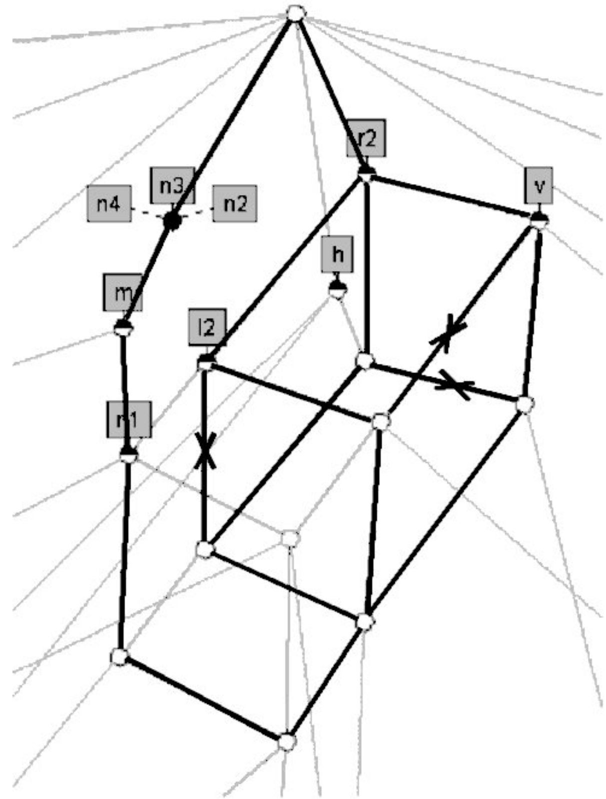
Criterion of Kuratowski:
A graph is planar iff it has neither K_5 nor $K_{3,3}$ as a *minor*.



K^5



$K_{3,3}$



part of the Hasse-diagram for Pāṇini's pratyāhāras

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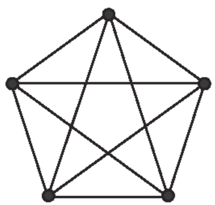
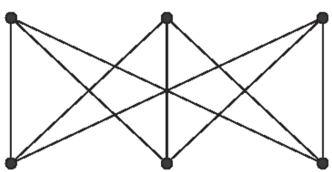


K^5



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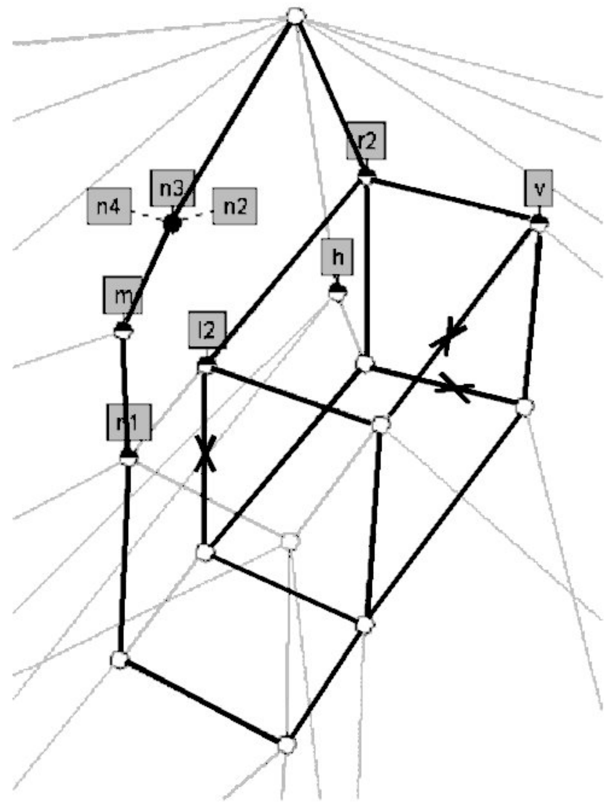
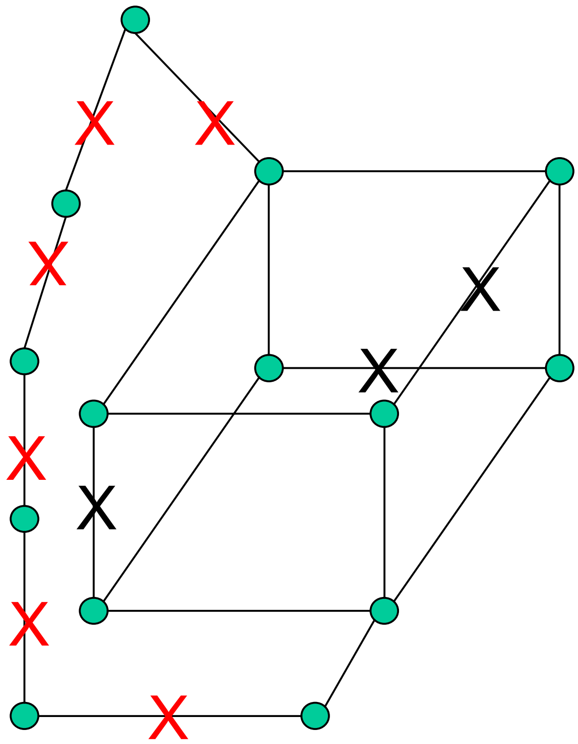


part of the Hasse-diagram for Pāṇini's pratyāhāras

K5 is a minor of the Ḥasse-diagram for the pratyāhāras



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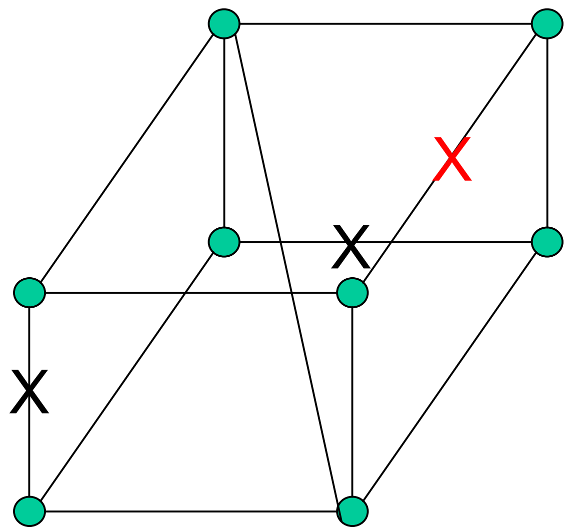

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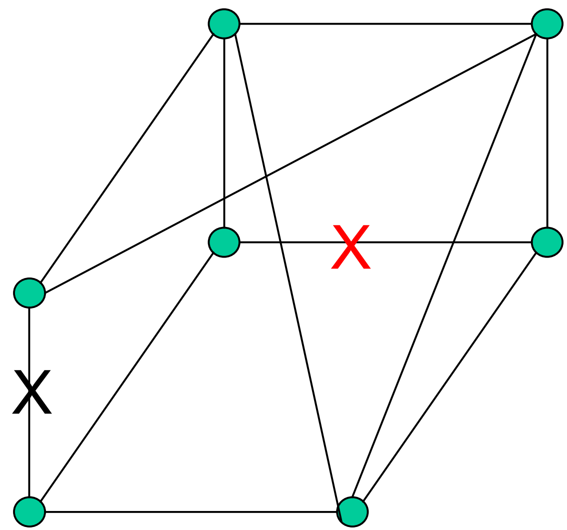

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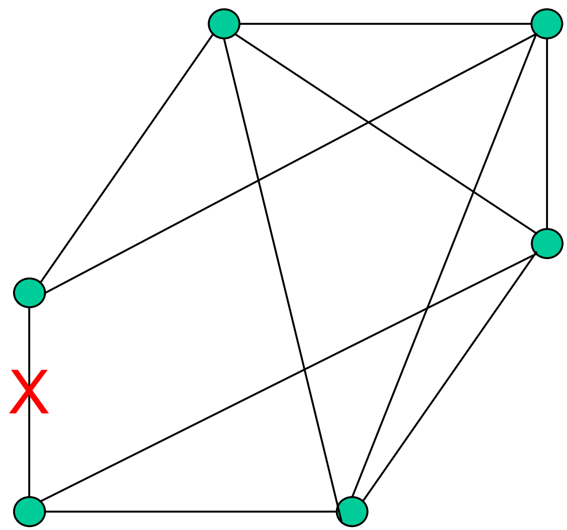

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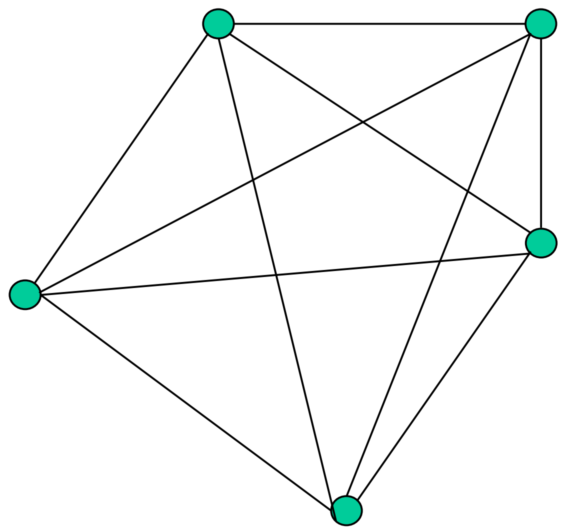
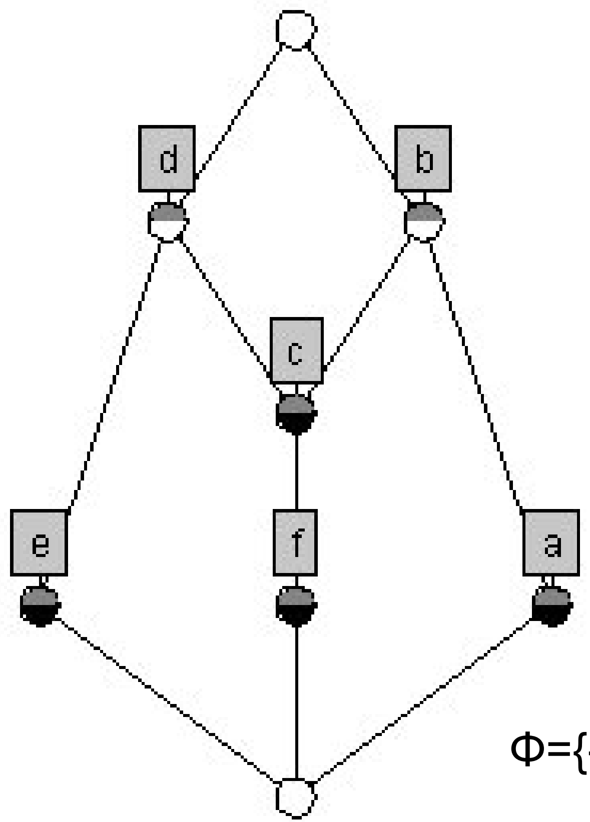


We are not done yet!



plane but not S-encodable!

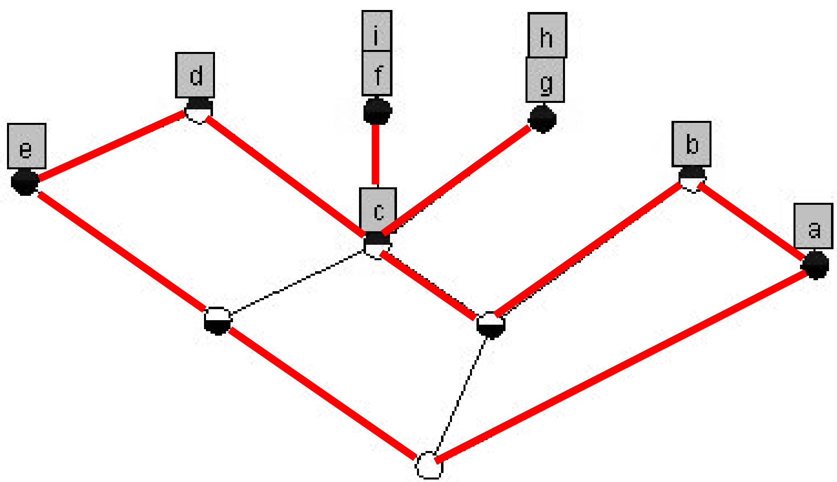
Φ={{d,e},{b,c,d,f,},{a,b},{b,c,d}}

Existence of S-alphabets

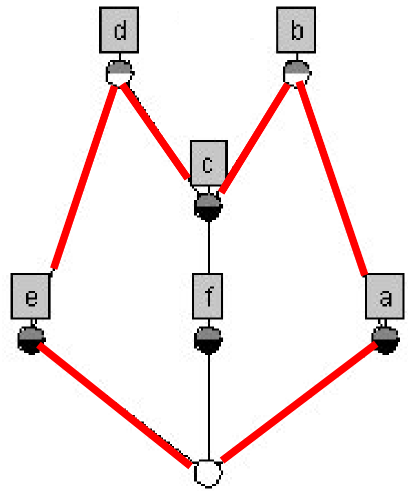
An S-alphabet of Φ exists iff

1. a plane Hasse-diagram for $(\mathcal{H}(\Phi),\supseteq)$ exists and

2. for every $b \in \mathcal{A}$ the S-graph contains a node labeled with b

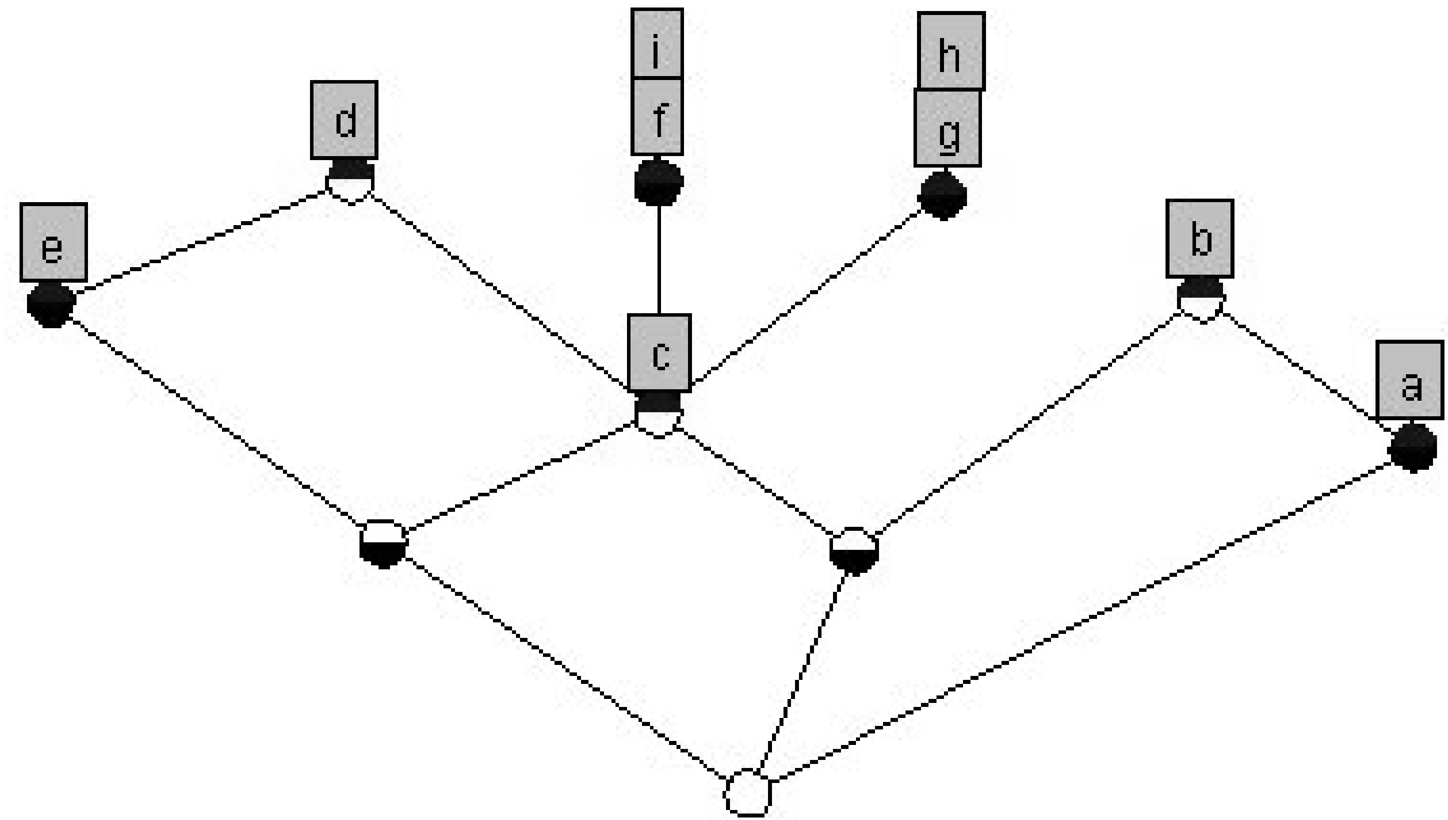


S-encodable



not S-encodable

Construction of S-alphabets



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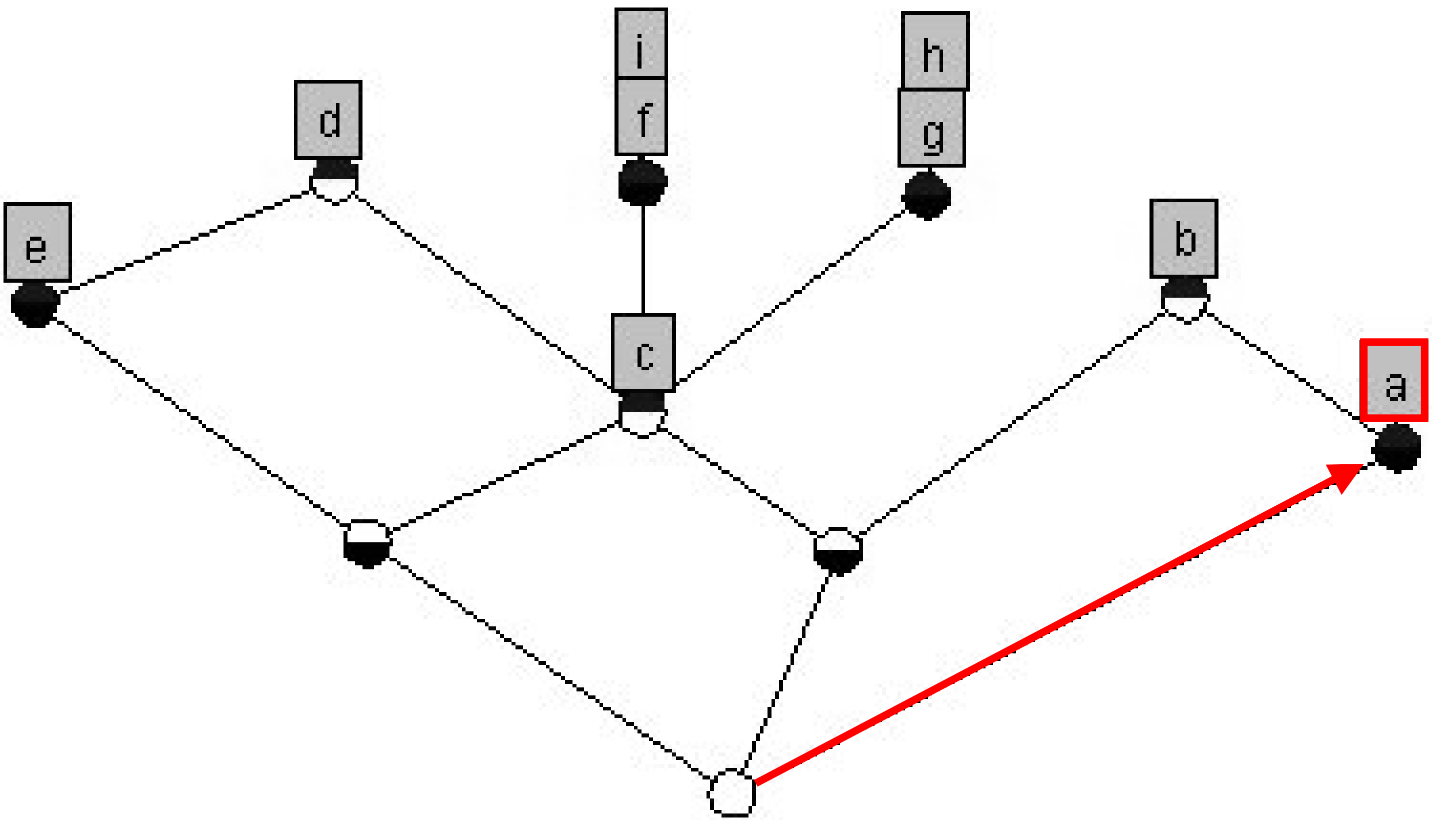

a

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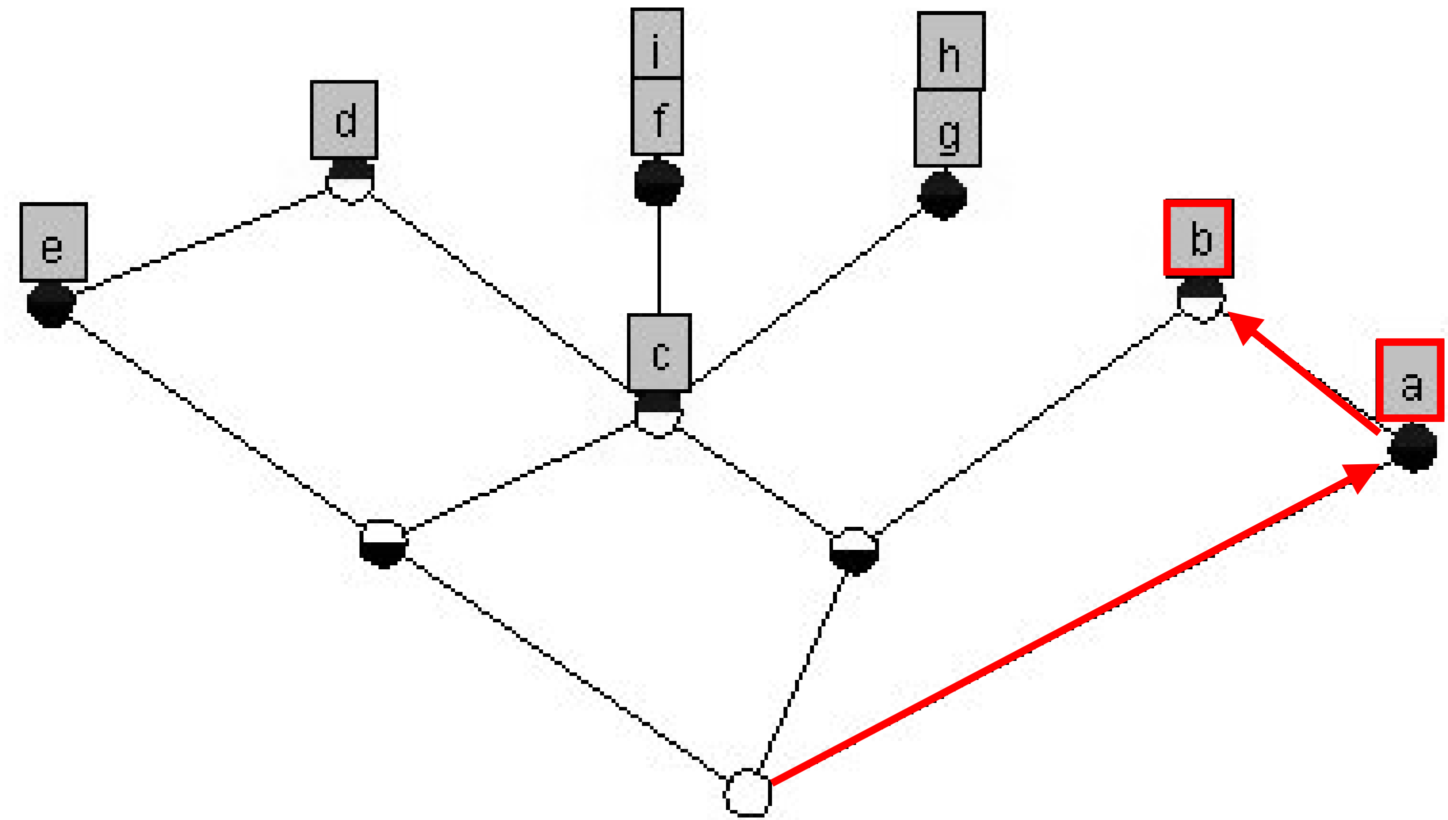

a b

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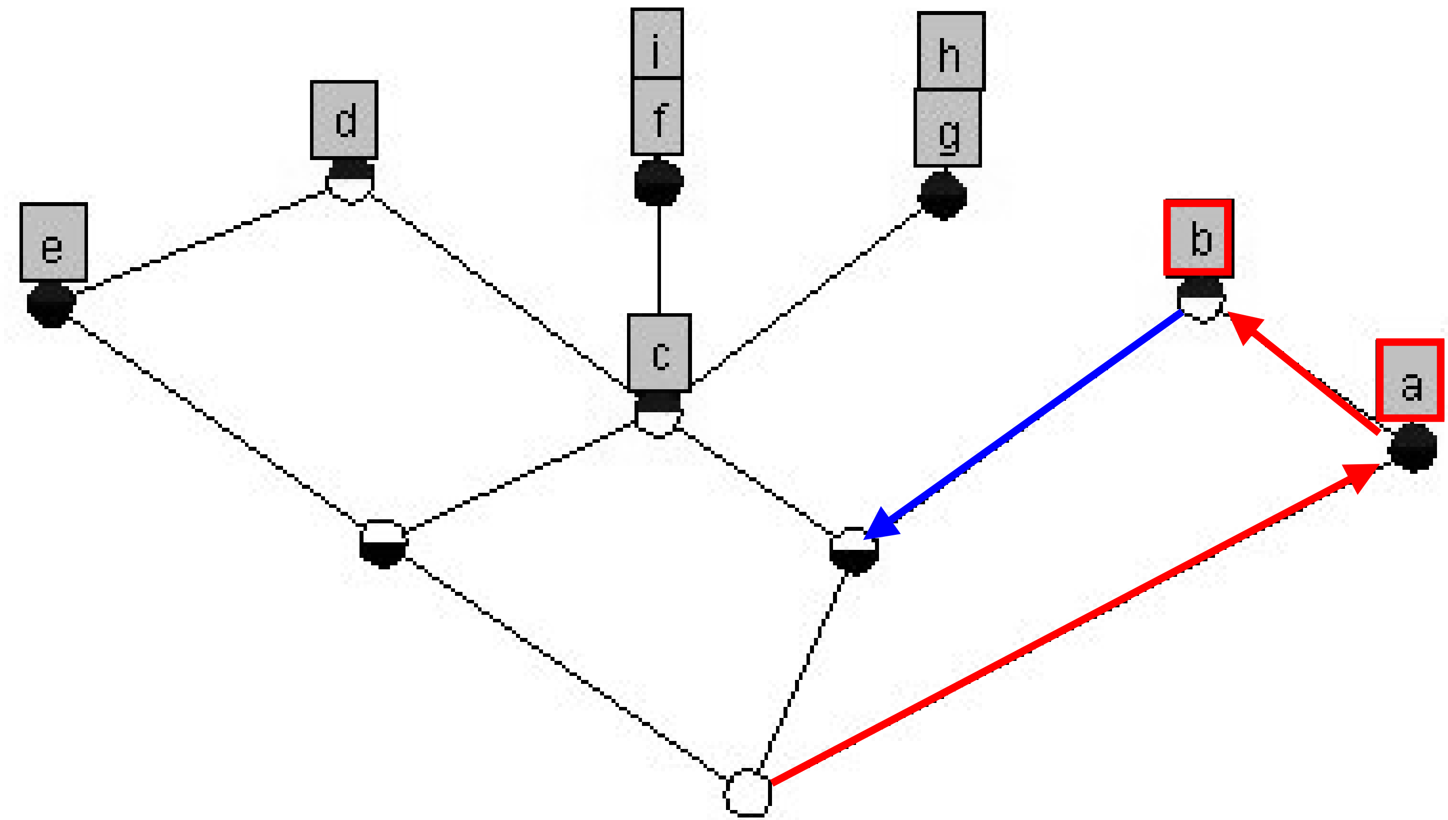

a b **M_1**

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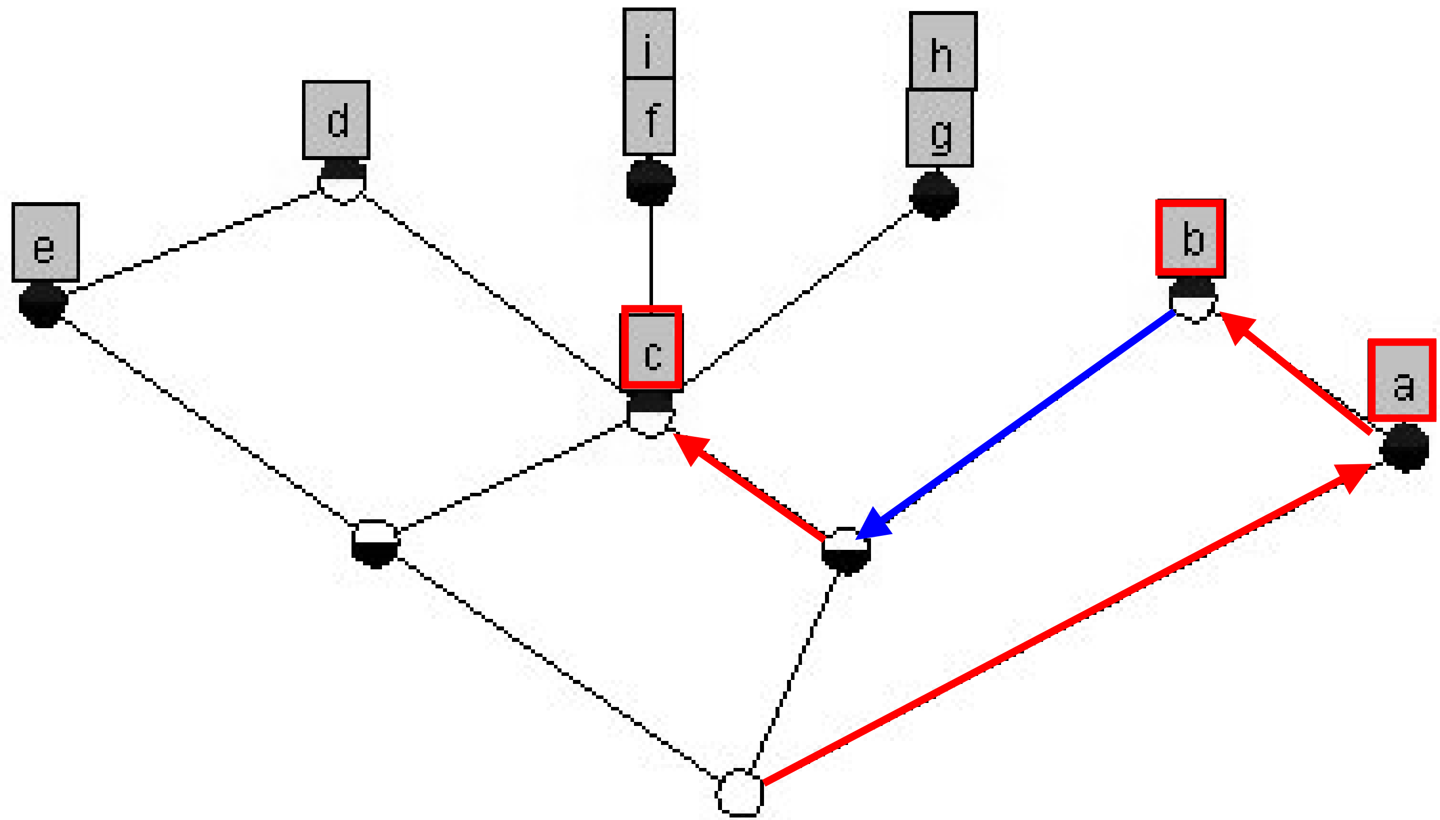

a b **M$_1$** c

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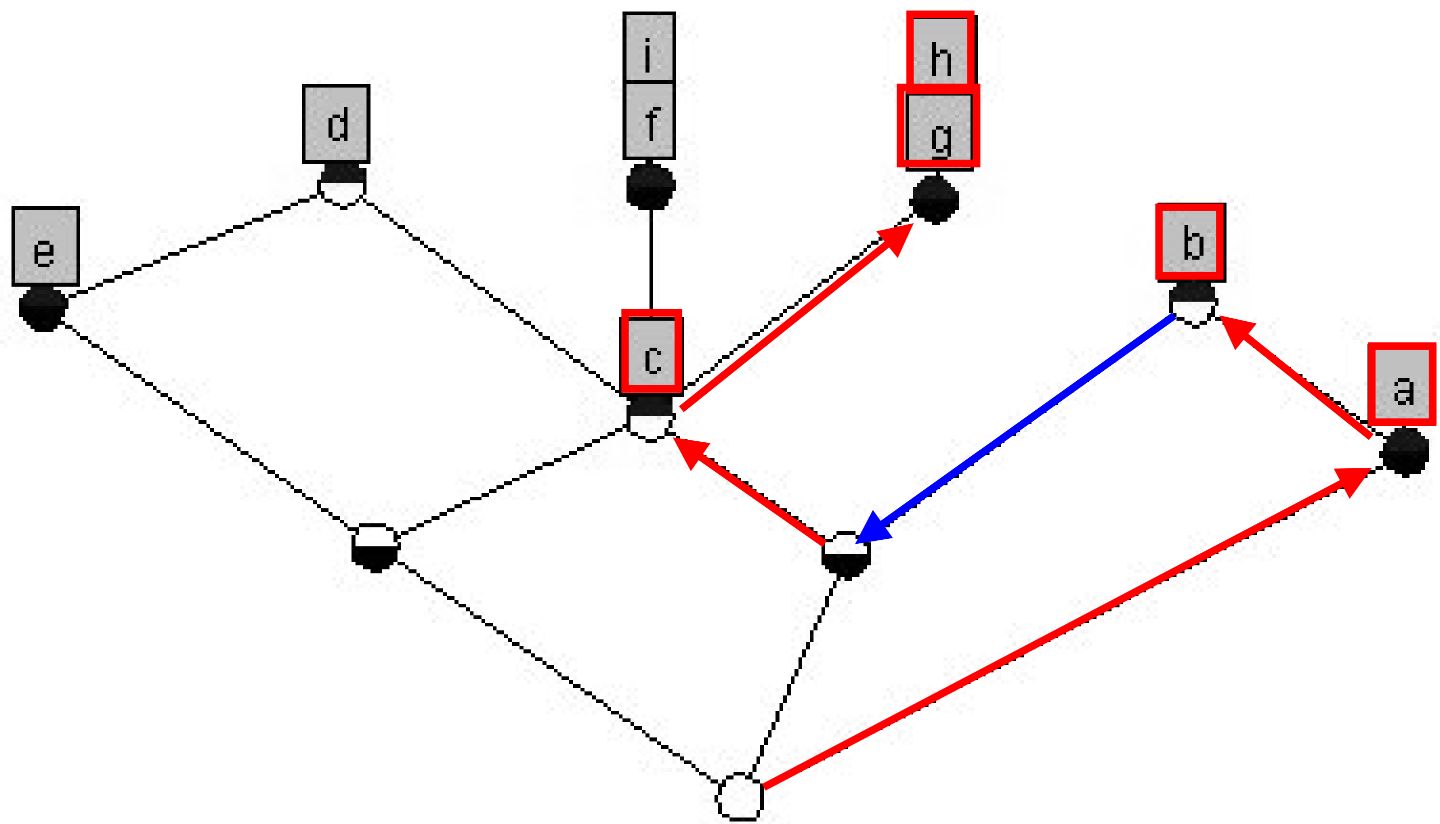
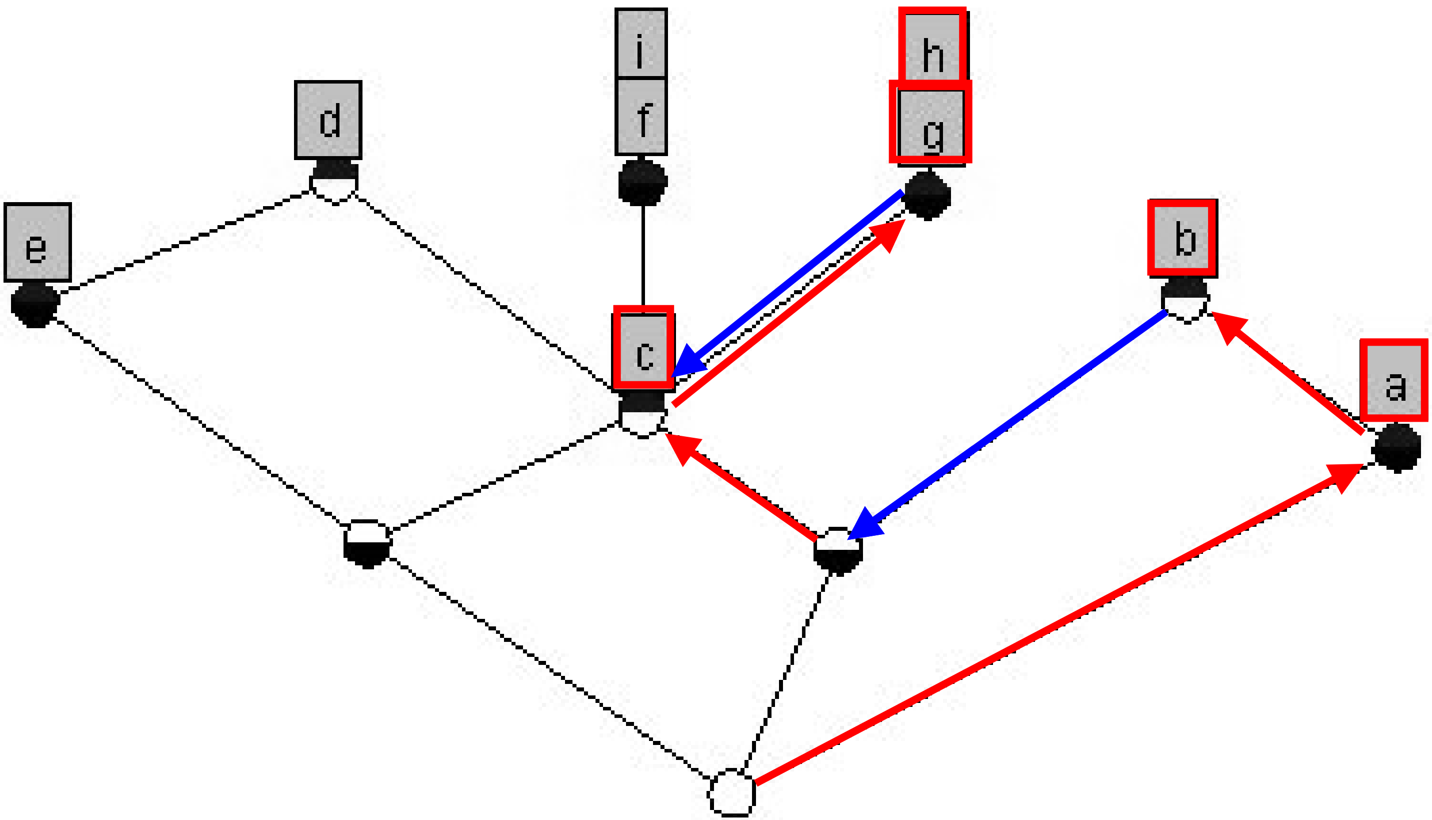


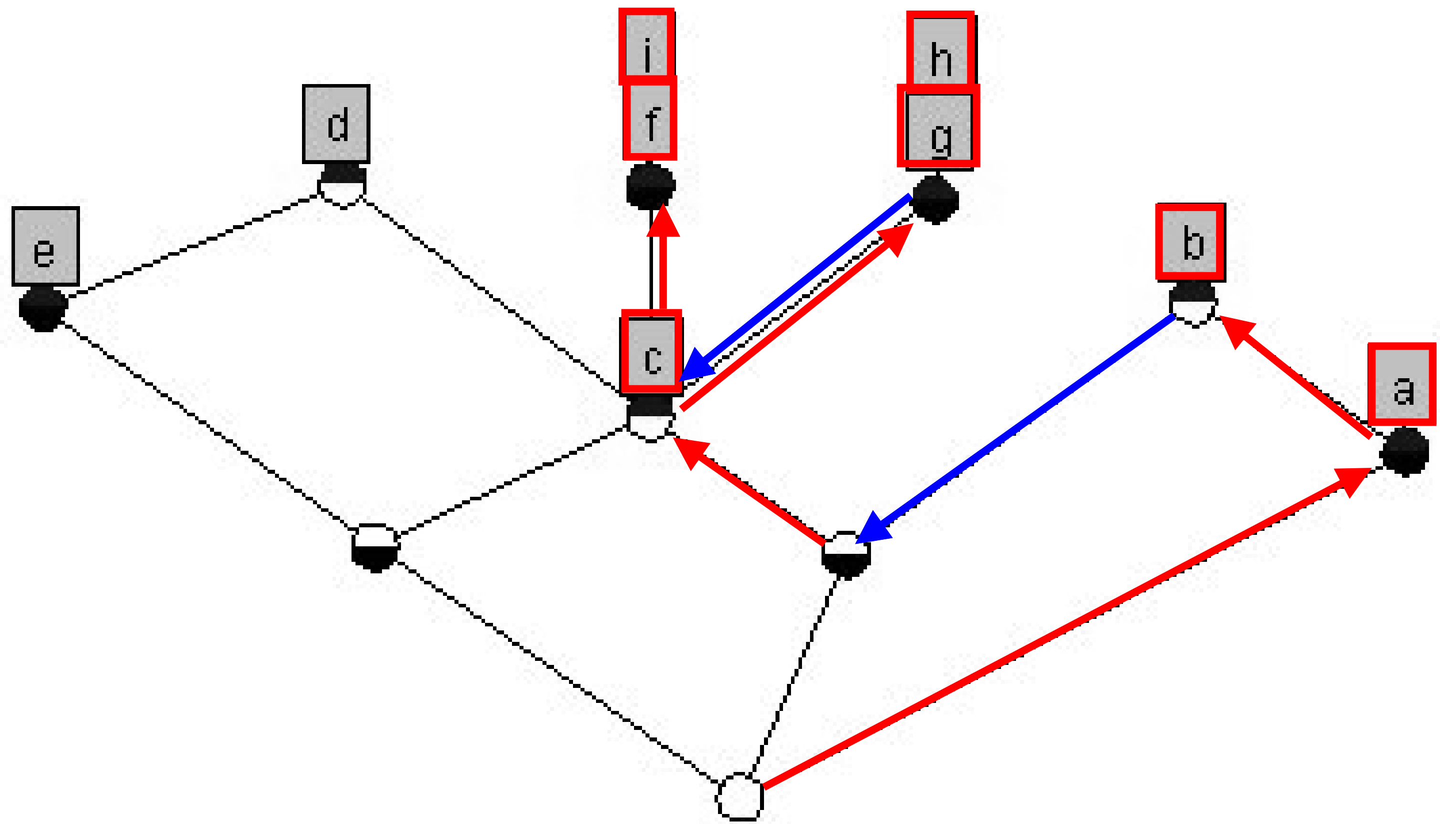
a b **M$_1$** c g h

Construction of S-alphabets



a b **M$_1$** c g h **M$_2$** c

Construction of S-alphabets



a b $\mathbf{M_1}$ c g h $\mathbf{M_2}$ c i f

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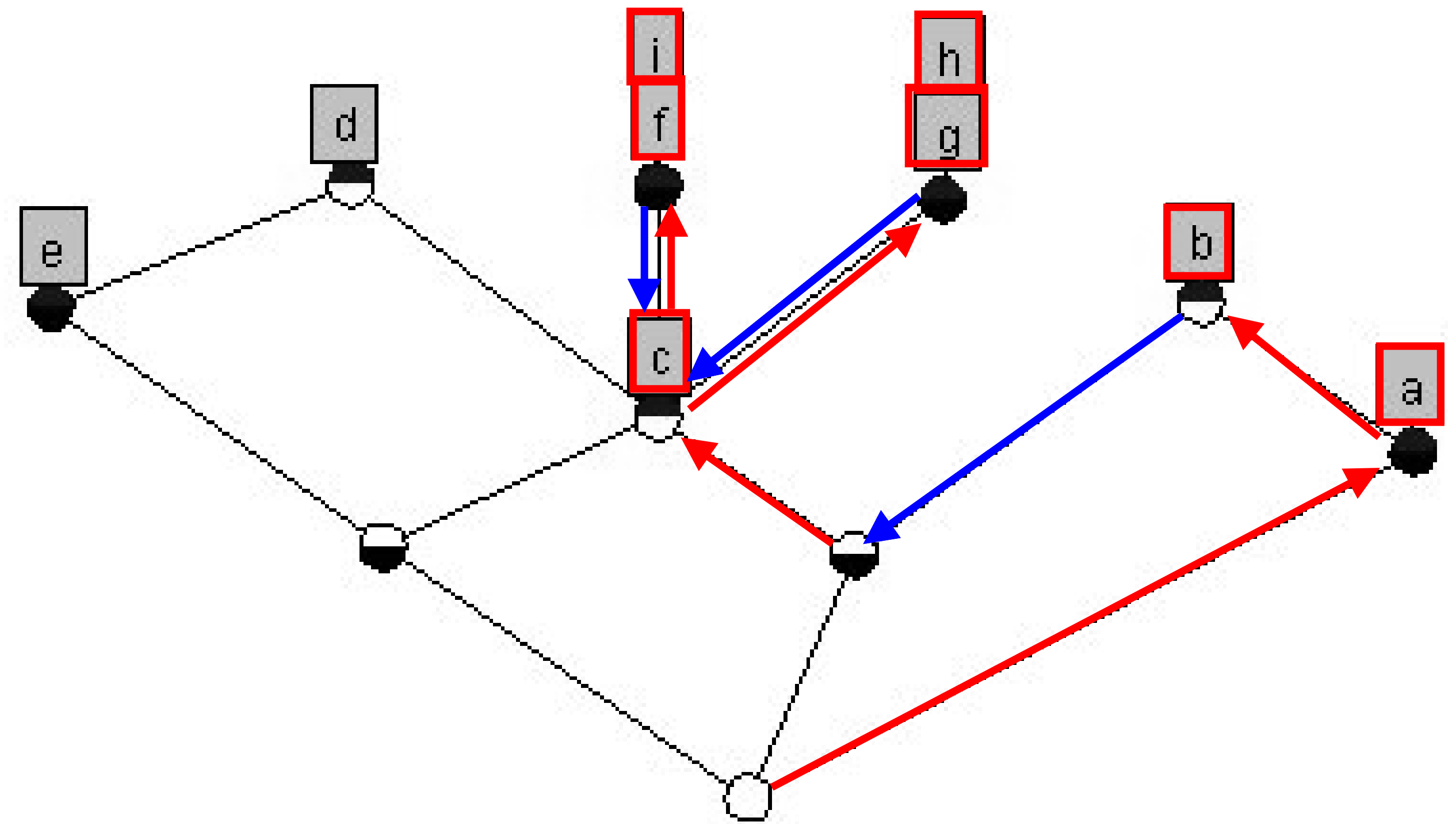
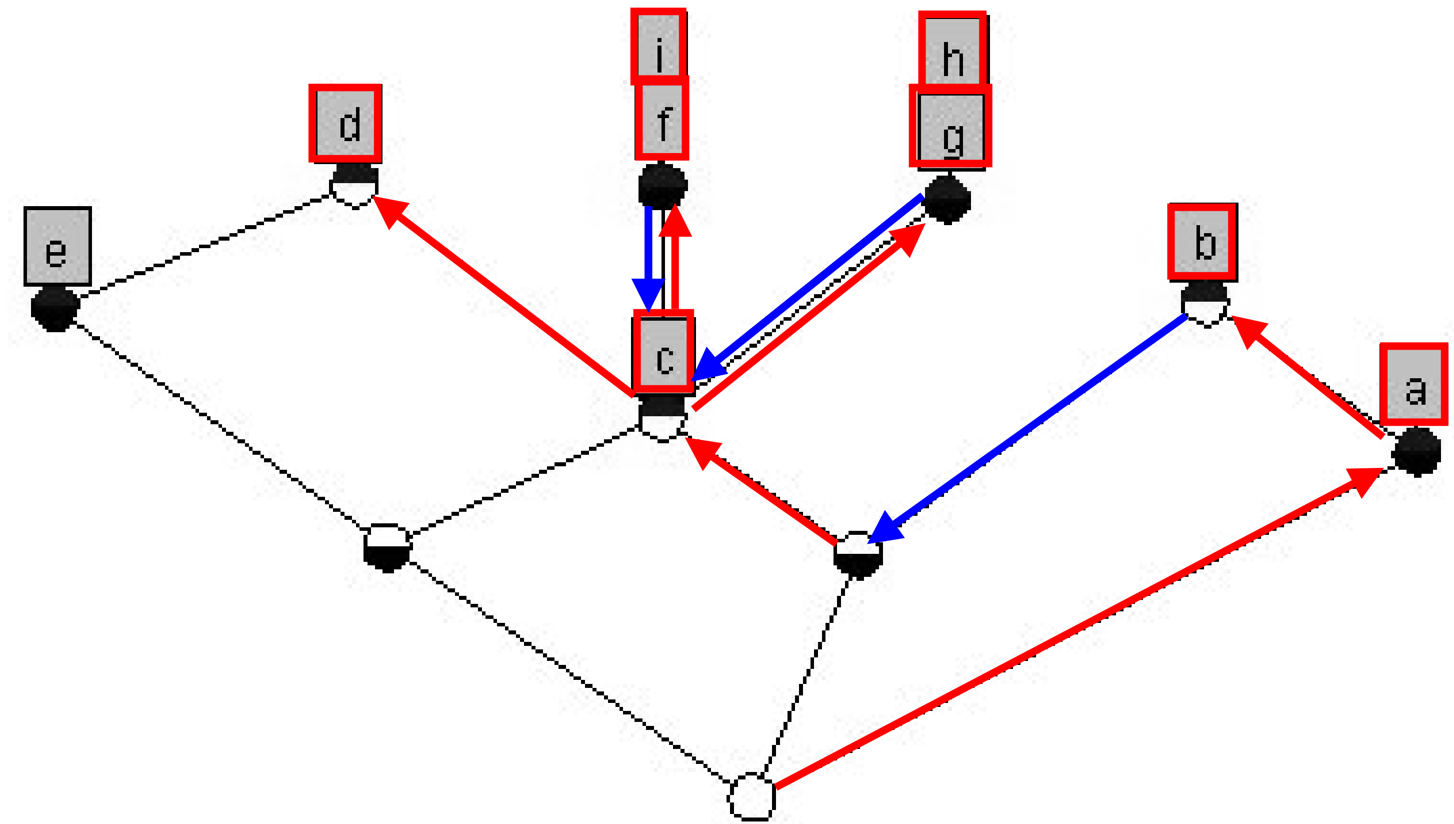


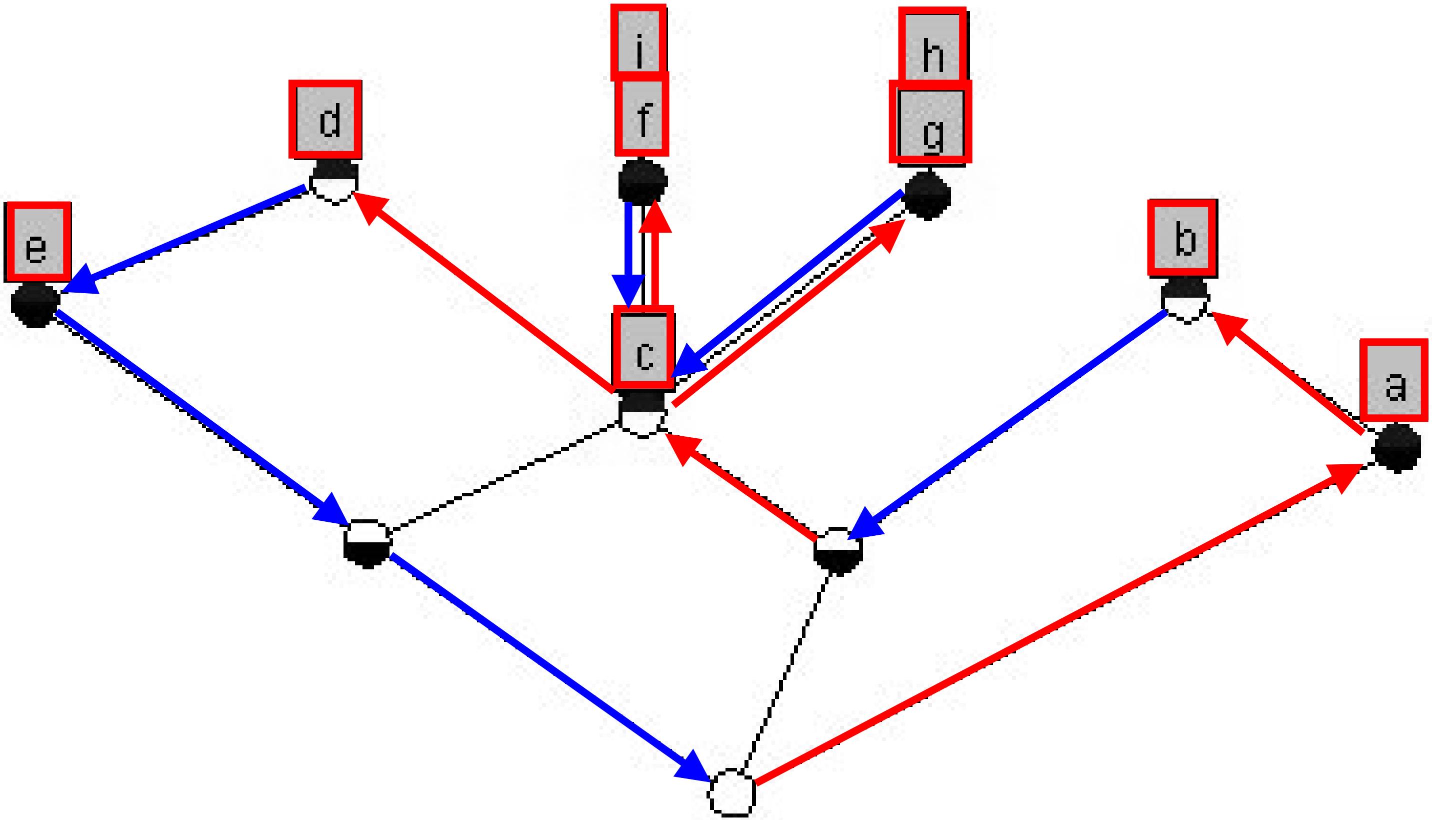
a b $\mathbf{M_1}$ c g h $\mathbf{M_2}$ c i f $\mathbf{M_3}$ c

Construction of S-alphabets



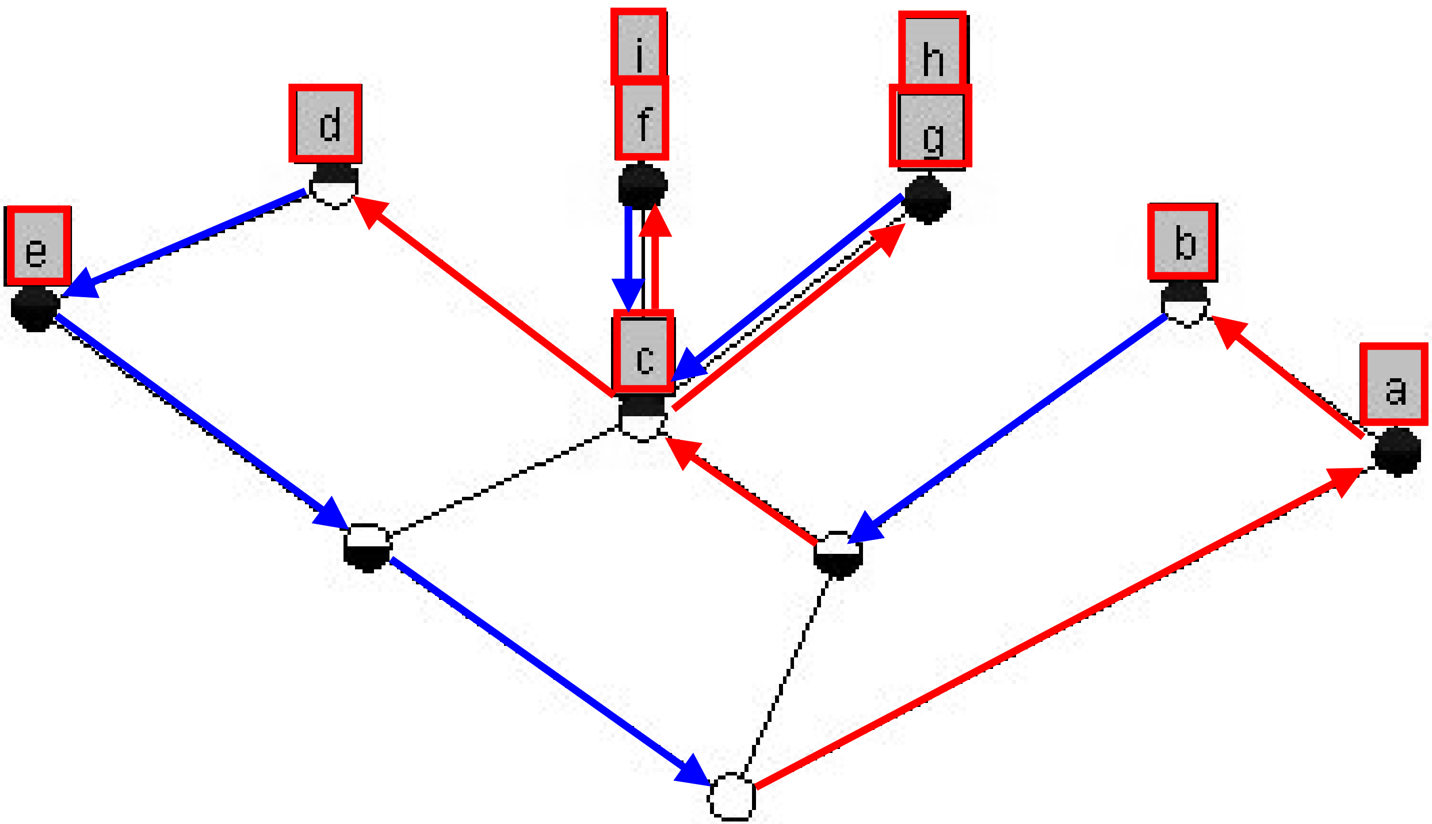
a b $\mathbf{M_1}$ c g h $\mathbf{M_2}$ c i f $\mathbf{M_3}$ c d

Construction of S-alphabets



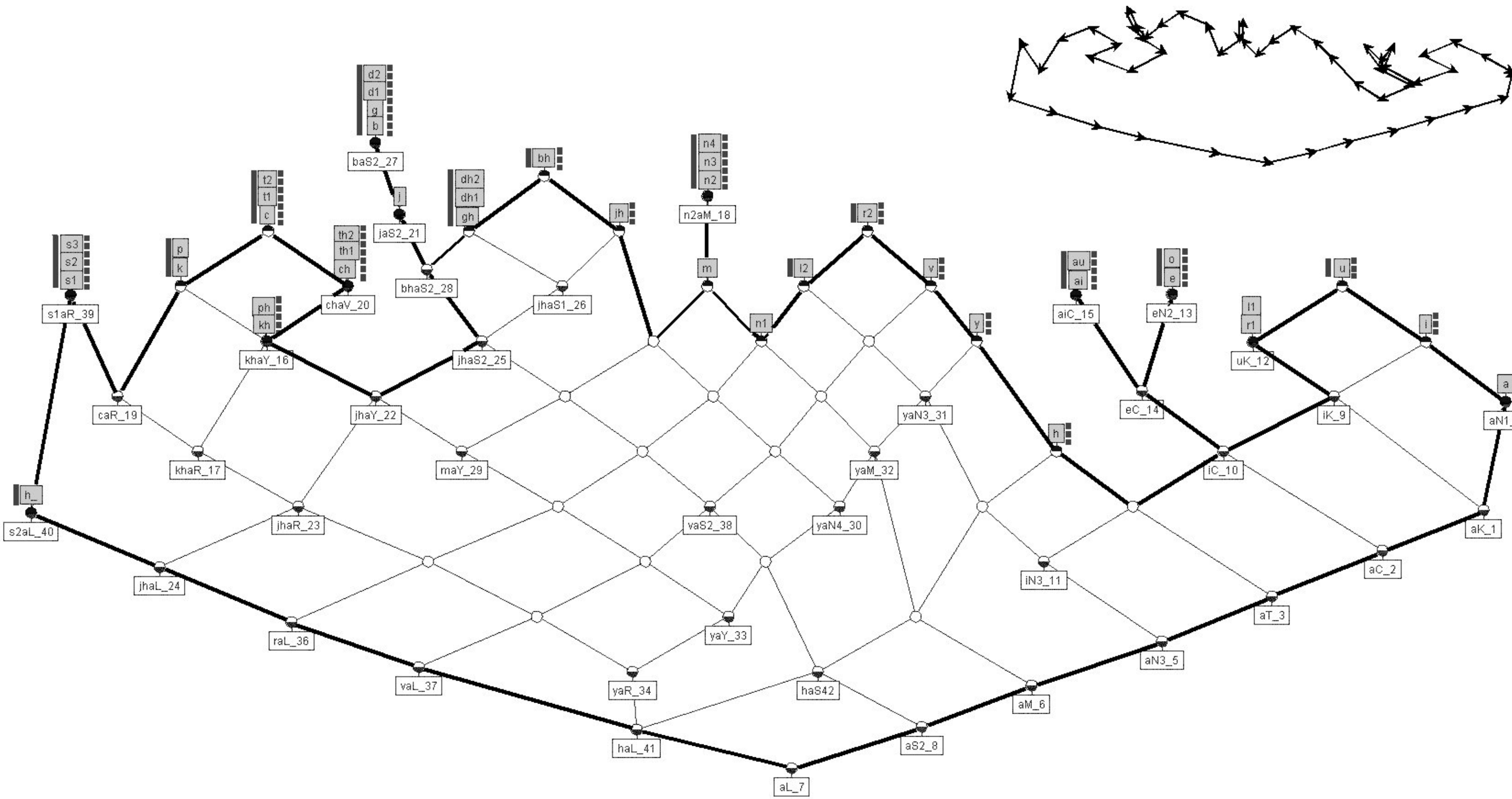
a b M_1 c g h M_2 c i f M_3 c d M_4 e M_5

Construction of S-alphabets



a b M_1 c g h M_2 ~~c~~ i f M_3 ~~c~~ d M_4 e M_5

Pāṇini's Śivasūtras are perfect



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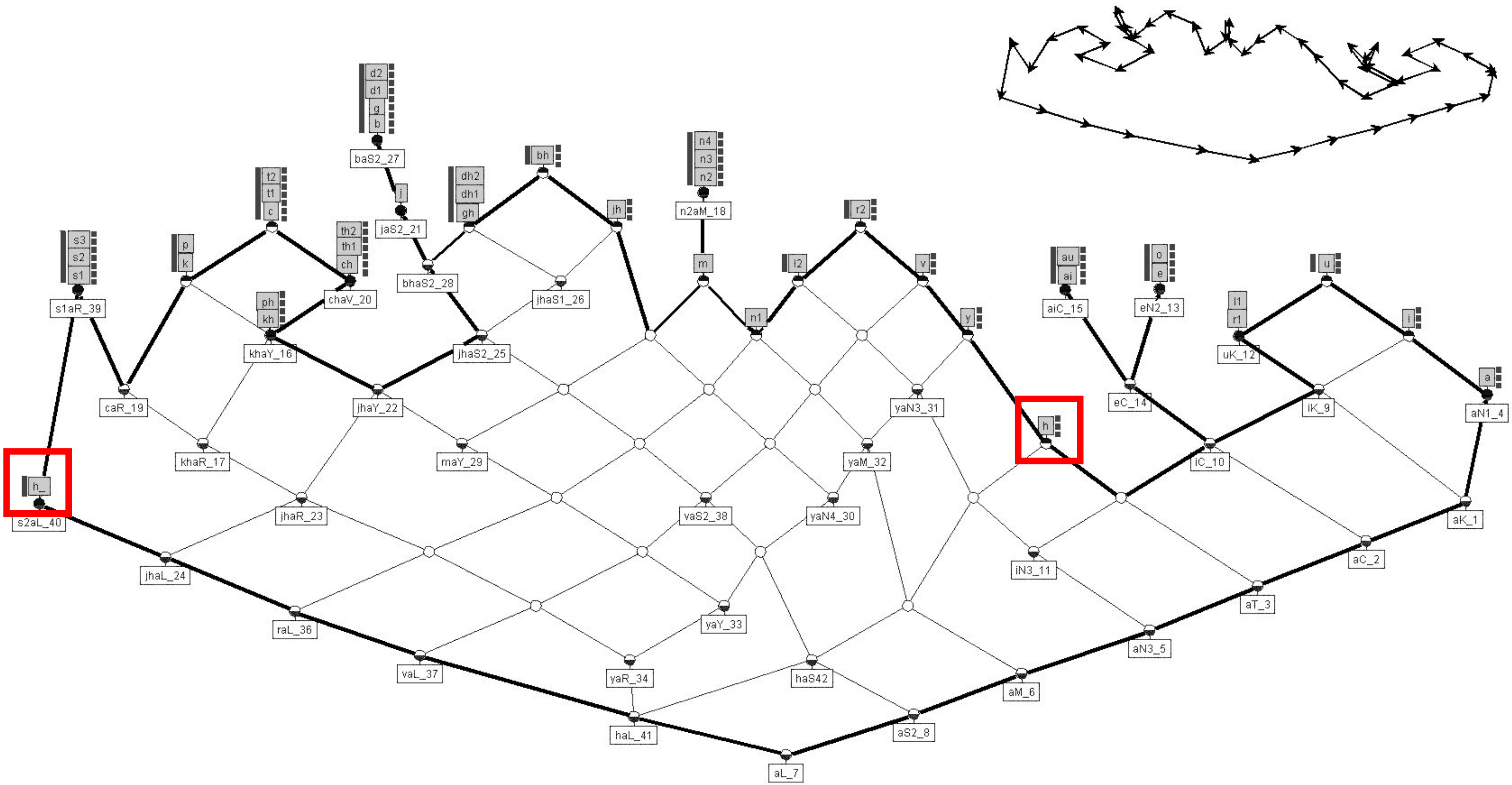

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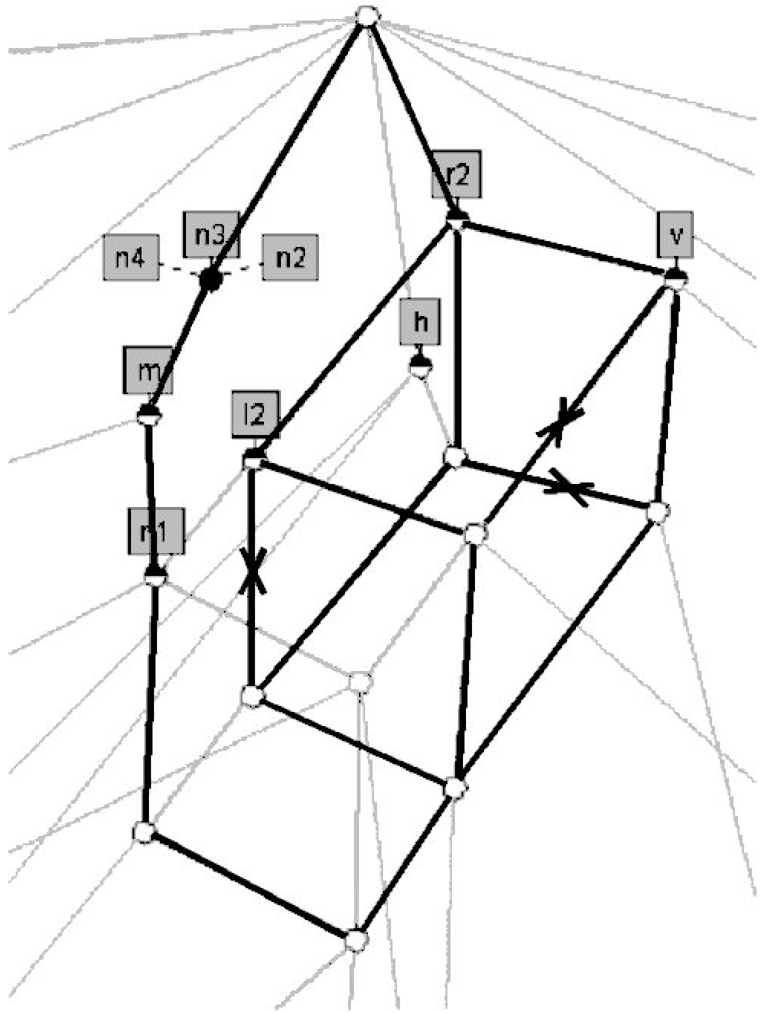
r2
n3
n4
n2
v
h
m
l2
n1

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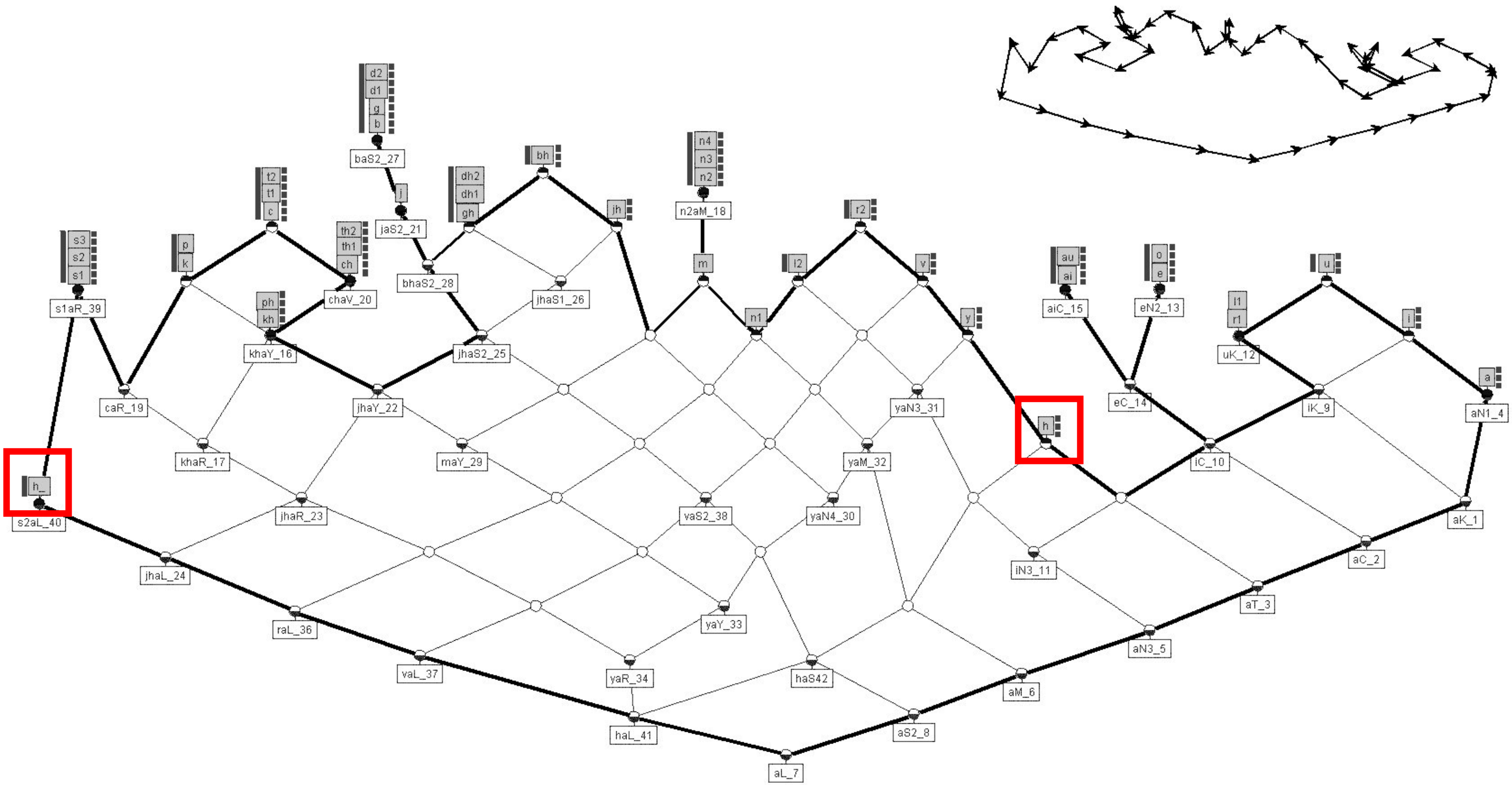

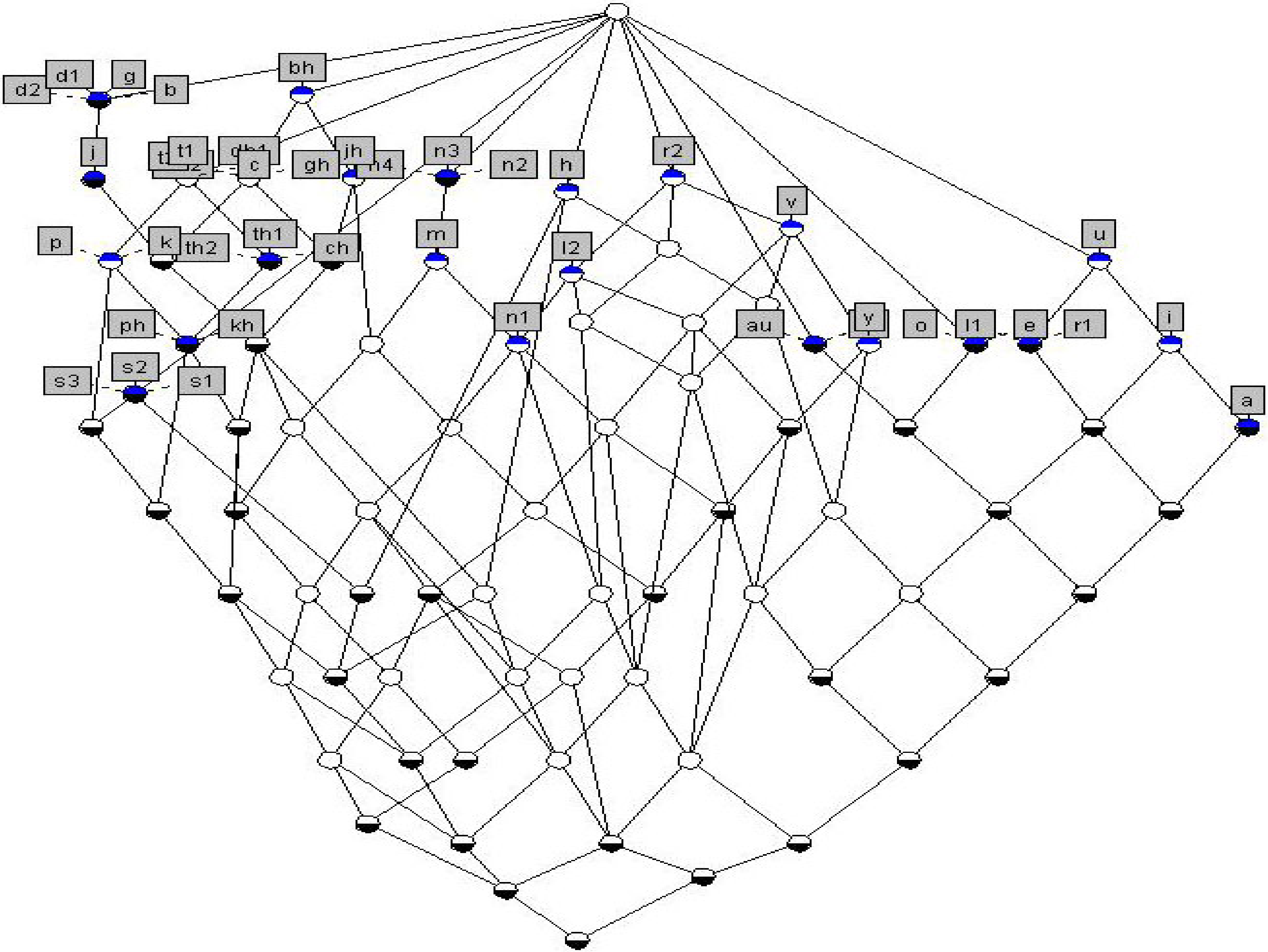
bh
d1
g
d2
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n4
n3
n2
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v
p
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th2
th1
ch
m
l2
u
ph
kh
n1
au
y
o
l1
e
r1
i
s3
s2
s1
a