On the Construction of Śivasūtra-Alphabets

Wiebke Petersen

Institute of Language and Information
University of Düsseldorf, Germany
petersew@uni-duesseldorf.de

IIT Bombay, 7th February 2009
**Phonological Rules**

**modern notation**

A is replaced by B if preceded by C and succeeded by D.

\[ A \rightarrow B / C \_ D \]

**example: final devoicing**

\[
\begin{bmatrix}
+ \text{ consonantal} \\
- \text{ nasal} \\
+ \text{ voiced}
\end{bmatrix}
\rightarrow
\begin{bmatrix}
+ \text{ consonantal} \\
- \text{ nasal} \\
- \text{ voiced}
\end{bmatrix}
\] /__#
Phonological Rules

**modern notation**

$A$ is replaced by $B$ if preceded by $C$ and succeeded by $D$.

$$A \rightarrow B/C_\_D$$

**example: final devoicing**

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+ & \text{consonantal} \\
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\end{bmatrix} / \_\_\#$$
Phonological Rules

**Modern notation**

A is replaced by B if preceded by C and succeeded by D.

\[
A \rightarrow B/C\_D
\]

**Panini’s linear Coding**

A + genitive, B + nominative, C + ablative, D + locative.

**Example**

- *sutra* 6.1.77: *iko yaṇacī* (इको यणचि)
- analysis: *[ik]*\textsubscript{gen}*[yaṇ]*\textsubscript{nom}*[ac]*\textsubscript{loc}
- modern notation: *[iK]* $\rightarrow$ *[yN]*$/\_$/*[aC]*
## Phonological Rules

### Modern notation

*A* is replaced by *B* if preceded by *C* and succeeded by *D*.

\[
A \rightarrow B/C\_D
\]

### Panini's linear Coding


### Example

- *sūtra 6.1.77: iko yañaci*  (इको यणचि)
- analysis: [ik]_{gen}[yañ]_{nom}[ac]_{loc}
- modern notation: [iK] → [yN]/_ [aC]
Pāṇini faced the problem of giving a linear representation of the nonlinear system of sound classes.

A similar problem occurs in . . .
Libraries
Warehouses and stores
### Pāṇini’s solution: Śivasūtras

<table>
<thead>
<tr>
<th></th>
<th>a i u</th>
<th>N</th>
<th>a·i·uṇ</th>
<th>r·lk</th>
</tr>
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<tr>
<td>2.</td>
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<td>jabagaḍadaś</td>
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<td>kh ph ch ṭh th c ṭ t</td>
<td>V</td>
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<td>śaṣasaḥ</td>
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On the Construction of Śivasūtra-Alphabets

Wiebke Petersen
# Pāṇini’s solution: Śivasūtras

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a·i·un | r·lk

एओड्। ऐओच्।  
e·oṅ | ai·auc

हयवर्ट। लण्।  
hayavaraṇ | laṇ

नम्मणनम्। ञम्म।  
ñamaññañanam | jhabhaṅ

घढघष्। जबगडदश्।  
ghadhadhas | jabagadadaś

खफ्फङ्घथचटतव्।  
kaphachathathacatatav

कपय्। शषसर्। हल्।  
kapay | šaṣasar | hal
## Pāṇini’s solution: Śivasūtras

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### Markers

- a·i·uṅ | r·l̄k
- e·oṅ | a·i·auc
- hayavaraṭ | laṅ
- ŋamaṇaṇaṇaṇam | jhabhaṅ
- ghaḍhadhaṣ | jabaḍḍadaś
c k h a p a ṭ h a ṭ a ṭ a ṭ a v | kapay | śaṣaśaṅ | hal
### Pratyāhāras

1. a i u ṇ
2. r ! K
3. e o Ń
4. ai au C
5. h y v r Ŵ
Pratyāhāras

1. a i u N
2. r l K
3. e o N
e o Ń
4. ai au C
5. h y v r T
   iK
Pratyāhāras

1. a i u N
2. r ! K
3. e o Ń
4. ai au C
5. h y v r Ź

iK = \langle i, u, r, ! \rangle
### Analysis of iko yaṇacī: \([iK] \rightarrow [yŅ]/\_ [aC]\)

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**Notes:**
- \([iK] \rightarrow [yŅ]/\_ [aC]\)
- \(\langle i, u, ɾ, !_ \rangle \rightarrow \langle y, v, r, l \rangle/\_ \langle a, i, u, ɾ, !_ , e, o, ai, au \rangle\)
Analysis of iko yaṇaci: \([iK] \rightarrow [yN]/\_ [aC]\)

1. \(a\ i\ u\ N\)
2. \(r\ !\ K\)
3. \(e\ o\ Ñ\)
4. \(ai\ au\ C\)
5. \(h\ y\ v\ r\ T\)
6. \(l\ Ñ\)

\([iK] \rightarrow [yN]/\_ [aC]\)
\(\langle i, u, r, l\rangle \rightarrow \langle y, v, r, l\rangle/\_ \langle a, i, u, r, l, e, o, ai, au\rangle\)
## General problem of S-sortability

Given a set of classes, order the elements of the classes (without duplications) in a linear order (in a list) such that each single class forms a continuous interval with respect to that order.

- The target orders are called **S-orders**
- A set of classes is **S-sortable** if it has an S-order
General problem of Śivasūtra-alphabets (S-alphabets)

Given a set of classes, find an S-order of the elements of the classes. Interrupt this list by markers such that each single class can be denoted by a sound-marker-pair (*pratyāhāra*).

Note that every S-order becomes a Śivasūtra-alphabet (S-alphabet) by adding a marker behind each element.

Given the set of classes \(\{\{a, b\}, \{a, b, c\}, \{a, b, c, d\}\}\), the order \(a\ b\ c\ d\) is one of its S-orders and \(a\ M_1\ b\ M_2\ c\ M_3\ d\ M_4\) is one of its S-alphabets.
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Some more Examples

**S-sortable example**

The set of classes:
\[
\{\{d, e\}, \{a, b\}, \{b, c, d, f, g, h, i\}, \{f, i\}, \{c, d, e, f, g, h, i\}, \{g, h\}\}
\]

is S-sortable;

one of its S-orders is
\[
a b c g h f i d e
\]

**non-S-sortable example**

The set of classes:
\[
\{\{a, b\}, \{b, c\}, \{a, c\}\}
\]

is not S-sortable.

**non-S-sortable example**

The set of classes:
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\{\{d, e\}, \{a, b\}, \{b, c, d\}, \{b, c, d, f\}\}
\]

is not S-sortable.
Some more Examples

S-sortable example
The set of classes:
\[ \{\{d, e\}, \{a, b\}, \{b, c, d, f, g, h, i\}, \{f, i\}, \{c, d, e, f, g, h, i\}, \{g, h\}\} \]

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\[ a \ b \ c \ g \ h \ f \ i \ d \ e \]

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Some more Examples

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Some more Examples

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Some more Examples

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Some more Examples

**S-sortable example**

The set of classes:
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non-S-sortable example

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\]
is not S-sortable.
\[
a \ b \ c \ d \ e \text{ or } e \ d \ c \ b \ a
\]
Some more Examples

S-sortable example

The set of classes:
\[\{\{d, e\}, \{a, b\}, \{b, c, d, f, g, h, i\}, \{f, i\}, \{c, d, e, f, g, h, i\}, \{g, h\}\}\]

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non-S-sortable example

The set of classes:
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non-S-sortable example

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\[a \ b \ c \ d \ e\]
or
\[e \ d \ c \ b \ a\]
Visualize relations

\{\{d, e\}, \{a, b\}, \{b, c, d, f, g, h, i\}, \{f, i\}, \{c, d, e, f, g, h, i\}, \{g, h\}\}
Visualize relations

\{ \{d, e\}, \{a, b\}, \{b, c, d, f, g, h, i\}, \{f, i\}, \{c, d, e, f, g, h, i\}, \{g, h\} \}

concept lattice
Visualize relations

\{\{d, e\}, \{a, b\}, \{b, c, d, f, g, h, i\}, \\
\{f, i\}, \{c, d, e, f, g, h, i\}, \{g, h\}\} 

\{\{a, b\}, \{b, c\}, \{a, c\}\} 

\{\{d, e\}, \{a, b\}, \{b, c, d, f\}\}
Main theorem of S-sortability

A set of classes is S-sortable without duplications if one of the following equivalent statements is true:

1. Its concept lattice is Hasse-planar and for any element \( a \) there is a node labeled \( a \) in the S-graph.

2. The concept lattice of the enlarged set of classes is Hasse-planar.

3. The Ferrers-graph of the enlarged set of classes is bipartite.
Main theorem of S-sortability

A set of classes is S-sortable without duplications if one of the following equivalent statements is true:

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2. The concept lattice of the enlarged set of classes is Hasse-planar.

3. The Ferrers-graph of the enlarged set of classes is bipartite.

```
[Diagram of Hasse-planar concept lattice with nodes labeled a, b, c, d, e, f, and edges connecting them in a bipartite manner.]
```

```
[Diagram of a not S-sortable example with a similar structure but not meeting the bipartite condition.]
```
Hasse-planarity

\{\{a, b\}, \{a, c\}, \{b, c\}\}

planar, but not Hasse-planar
2nd condition: Hasse-planar $\Rightarrow$ S-sortable

\[
\{\{d, e\}, \{a, b\}, \{b, c, d, f, g, h, i\}, \{f, i\}, \{c, d, e, f, g, h, i\}, \{g, h\}\}
\]
2nd condition: S-sortable $\Rightarrow$ Hasse-planar

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Wiebke Petersen
2nd condition: evaluation

- It is of no help in the construction of S-alphabets with minimal number of markers.
- The planarity of a graph is difficult to check.
Main theorem of S-sortability

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3. The Ferrers-graph of the enlarged set of classes is bipartite.
1st condition ⇔ 2nd condition
S-alphabets with a minimal number of markers

procedure

Start with the empty sequence and choose a walk through the S-graph:

- While moving upwards do nothing.
- While moving downwards along an edge add a new marker to the sequence unless its last element is already a marker.
- If a labeled node is reached, add the labels in arbitrary order to the sequence, unless it has been added before.
procedure

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S-alphabets with a minimal number of markers

**procedure**

Start with the empty sequence and choose a walk through the S-graph:

- While moving upwards do nothing.
- While moving downwards along an edge add a new marker to the sequence unless its last element is already a marker.
- If a sound is reached, add the sound to the sequence, unless it has been added before.
S-alphabets with a minimal number of markers

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- If a sound is reached, add the sound to the sequence, unless it has been added before.

\[ edM_1cfi \]
S-alphabets with a minimal number of markers

procedure

Start with the empty sequence and choose a walk through the S-graph:

- While moving upwards do nothing.
- While moving downwards along an edge add a new marker to the sequence unless its last element is already a marker.
- If a sound is reached, add the sound to the sequence, unless it has been added before.
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$$ed M_1 cfi M_2 gh M_3$$
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\[ ed M_1 cfi M_2 gh M_3 b M_4 a \]
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ed \ M_1 c f i M_2 g h M_3 b M_4 a M_5
1st condition: evaluation

+ Allows the construction of S-alphabets with minimal number of markers.
  - The planarity of a graph is difficult to check.
Main theorem of S-sortability

A set of classes is S-sortable without duplications if one of the following equivalent statements is true:

1. Its concept lattice is Hasse-planar and for any element \( a \) there is a node labeled \( a \) in the S-graph.

2. The concept lattice of the enlarged set of classes is Hasse-planar.

3. The Ferrers-graph of the enlarged set of classes is bipartite.

- The Ferrers-graph can be computed directly from the set of classes.
- Its bipartity can be checked algorithmically.
Theorem (Zschalig 2007)

The concept lattice of a formal context is Hasse-planar if and only if its Ferrers-graph is bipartite.

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
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</tr>
</thead>
<tbody>
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<td>x</td>
<td>x</td>
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<td>x</td>
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</tbody>
</table>
3rd condition: terminology & proof

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3rd condition: example

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<td>3</td>
<td>×</td>
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</tr>
</tbody>
</table>

On the Construction of Śivasūtra-Alphabets

Wiebke Petersen
3rd condition: example

On the Construction of Śivasūtra-Alphabets  Wiebke Petersen
3rd condition: evaluation

- It is of no help in the construction of S-alphabets with minimal number of markers.
- It can be checked easily by an algorithm.
Getting back to Pāṇini’s problem

Q: Are the Śivasūtras minimal (with respect to length)?
What does minimal mean?

The Śivasūtras are **not minimal** if it is possible to rearrange the Sanskrit sounds in a new list with markers such that

1. each *pratyāhāra* forms an interval ending before a marker,
2. no sound occurs twice

**or** one sound occurs twice but less markers are needed.

⇒ duplicating a sound is worse than adding markers
<table>
<thead>
<tr>
<th>Introduction</th>
<th>Pratyāhāras</th>
<th>Generalization</th>
<th>Main theorem</th>
<th>Minimality of Śivasūtras</th>
</tr>
</thead>
</table>

Are Pāṇini’s Śivasūtras minimal?
Are Pāṇini’s Śivasūtras minimal?

is it necessary to duplicate a sound?
Are Pāṇini’s Śivasūtras minimal?

if no

is it necessary to duplicate a sound?

if yes

is it the best choice to duplicate ‘h’?

Śivasūtras are not minimal
Are Pāṇini’s Śivasūtras minimal?

- Is it necessary to duplicate a sound?
  - No
  - Yes

- Is it the best choice to duplicate 'h'?
  - No
  - Yes

- Given the duplication of 'h', is the number of anubandhas minimal?

Šivasūtras are not minimal
Are Pāṇini’s Śivasūtras minimal?

- Is it necessary to duplicate a sound?
  - No
  - Yes
    - Is it the best choice to duplicate 'h'?
      - No
      - Yes
        - Given the duplication of 'h', is the number of anubandhas minimal?
          - No
          - Yes
            - Śivasūtras are minimal
            - Śivasūtras are not minimal
Is it necessary to duplicate a sound?

**Main theorem on S-sortability (part 1a)**
If a set of classes is S-sortable, then its concept lattice is Hasse-planar.

concept lattice of Pāṇini’s pratyāhāras
Is it necessary to duplicate a sound?

**Criterion of Kuratowski**

A graph which has the graph as a minor is not planar.
Is it necessary to duplicate a sound?

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There is no S-alphabet for the set of classes given by Pāṇini’s *pratyāhāras* without duplicated elements!
Are Pāṇini’s Śivasūtras minimal?

is it necessary to duplicate a sound?

yes

is it the best choice to duplicate 'h'?
Altogether there exists 249 independent triples. $h$ is included in all of them.
Are Pāṇini’s Śivasūtras minimal?

- is it necessary to duplicate a sound? yes
- is it the best choice to duplicate ‘h’? yes

given the duplication of ‘h’, is the number of anubandhas minimal?
Concept lattice of Pāṇini’s *pratyāhāras* with duplicated $h$
Concept lattice of Pāṇini’s pratyāhāras with duplicated $h$
With the Śivasūtras Pāṇini has chosen one out of nearly 12 million minimal S-alphabets!
Are Pāṇini's Śivasūtras minimal?

- Is it necessary to duplicate a sound? (Yes)

- Is it the best choice to duplicate 'h'? (Yes)

- Given the duplication of 'h', is the number of anubandhas minimal? (Yes)

Śivasūtras are minimal.
Open problems

The story is much more intricate

- We have neither shown that Pāṇini’s technique for the representation of sound classes is optimal
- nor that he has used his technique in an optimal way.
  - not all sound classes are denoted by pratyāhāras
  - rules overgeneralize
  - sūtra 1.3.10: yathāsaṃkhyamanudeśaḥ samānāṁ
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  - *sūtra* 1.3.10: *yathāsaṃkhyamanudeśaḥ samānām*
\langle a, i, u, M_1, \{r, l\}_1, M_2, \langle\langle e, o\rangle_2, M_3\rangle, \langle\langle ai, au\rangle_3, M_4\rangle\rangle_4,

h, y, v, r, M_5, l, M_6, \= m, \{\= n, \= n, n\}_5, M_7, jh, bh, M_8,

\{gh, dh, dh\}_6, M_9, j, \{b, g, d, d\}_7, M_{10}, \{kh, ph\}_8, \{ch, \= th, th\}_9,

\{c, t, t\}_10, M_{11}, \{k, p\}_11, M_{12}, \{\= s, s, s\}_12, M_{13}, h, M_{14}\rangle

2! \times 2! \times 2! \times 2! \times 3! \times 3! \times 4! \times 2! \times 3! \times 3! \times 2! \times 3!

\{1\}_1 \{2\}_2 \{3\}_3 \{4\}_4 \{5\}_5 \{6\}_6 \{7\}_7 \{8\}_8 \{9\}_9 \{10\}_10 \{11\}_11 \{12\}_12

= 2 \times 2 \times 2 \times 2 \times 6 \times 6 \times 24 \times 2 \times 6 \times 6 \times 2 \times 6 = 11943936

On the Construction of Śivasūtra-Alphabets

Wiebke Petersen
Some numbers

- Pāṇini denotes 42 sound classes by *pratyāhāras*.
- The Śivasūtras allow the construction of 281 *pratyāhāras*.
- $2^{42} - 43 \ (> 2 \cdot 10^{12})$ possible sound classes.
- 11 (resp. 10, if unmarked classes are permitted) binary features are necessary to denote Pāṇini’s *pratyāhāras* ($\Rightarrow 2^{11} = 2048$, resp. $2^{10} = 1024$ classes can be constructed).
- Pāṇini has chosen 1 out of 11,943,936 minimal S-alphabets.
- The 42 sounds can be ordered in nearly $43! \ (> 6 \cdot 10^{52})$ lists in which $h$ occurs twice.


Origin of Pictures

- libraries (left):  
  http://www.meduniwien.ac.at/medizinischepsychologie/bibliothek.htm
- libraries (middle): http://www.math-nat.de/aktuelles/allgemein.htm
- libraries (right):  
  http://www.geschichte.mpg.de/deutsch/bibliothek.html
- warehouses:  
  http://www.metrogroup.de/servlet/PB/menu/1114920_l1/index.html
- stores: http://www.einkaufsparadies-schmidt.de/01bilder01/