On the Construction of *Śivasūtra*-Alphabets

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अइउण्। ऋऌक्। एओङ्। ऐऔच्। हयवरट्।
लण्। ञमङणनम्। झभञ्। घढधष्। जबगडदश्।
खफछठथचटतव्। कपय्। शषसर्। हल्।

Phonological Rules

modern notation

A is replaced by B if preceded by C and succeeded by D.

$$A \rightarrow B / _{C\_\_D}$$

example: final devoicing

$$\begin{bmatrix} + & \text{consonantal} \\ - & \text{nasal} \\ + & \text{voiced} \end{bmatrix} \rightarrow \begin{bmatrix} + & \text{consonantal} \\ - & \text{nasal} \\ - & \text{voiced} \end{bmatrix} / \_\_\sharp$$

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Phonological Rules

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Pāṇini's linear Coding

$$A + \text{genitive}, B + \text{nominative}, C + \text{ablative}, D + \text{locative}.$$

example

- *sūtra* 6.1.77: *iko yaṇaci* (इको यणचि)
- analysis: $[\text{ik}]_{\text{gen}}[\text{yaṇ}]_{\text{nom}}[\text{ac}]_{\text{loc}}$
- modern notation: [iK] → [yṆ]/__[aC]

Phonological Rules

modern notation

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Pāṇini faced the problem of giving a linear representation of the nonlinear system of sound classes.

A similar problem occurs in . . .

Libraries

Warehouses and stores

Pāṇini's solution: *Śivasūtras*

1.	a i u	Ṇ
2.	ṛ ḷ	K
3.	e o	Ṅ
4.	ai au	C
5.	h y v r	Ṭ
6.	l	Ṇ
7.	ñ m ṅ ṇ n	M
8.	jh bh	Ñ
9.	gh ḍh dh	Ṣ
10.	j b g ḍ d	Ś
11.	kh ph ch ṭh th c ṭ t	V
12.	k p	Y
13.	ś ṣ s	R
14.	h	L

अइउण्। ऋऌक्।
a·i·uṇ| *ṛ·ḷk*|
एओङ्। ऐऔच्।
e·oṅ| *ai·auc*|
हयवरट्। लण्।
hayavaraṭ| *laṇ*|
ञमङणनम्। झभञ्।
ñamaṅaṇanam| *jhabhañ*|
घढधष्। जबगडदश्।
ghaḍhadhaṣ| *jabagaḍadaś*|
खफछठथचटतव्।
khaphachaṭhathacaṭatav|
कपय्। शषसर्। हल्।
kapay| *śaṣasar*| *hal*|

Pāṇini's solution: *Śivasūtras*

1.	a i u	Ṇ
2.	ṛ ḷ	K
3.	e o	Ṅ
4.	ai au	C
5.	h y v r	Ṭ
6.	l	Ṇ
7.	ñ m ṅ ṇ n	M
8.	jh bh	Ñ
9.	gh ḍh dh	Ṣ
10.	j b g ḍ d	Ś
11.	kh ph ch ṭh th c ṭ t	V
12.	k p	Y
13.	ś ṣ s	R
14.	h	L

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8.	jh bh	Ñ
9.	gh ḍh dh	Ṣ
10.	j b g ḍ d	Ś
11.	kh ph ch ṭh th c ṭ t	V
12.	k p	Y
13.	ś ṣ s	R
14.	h	L
		markers

अइउण्। ऋऌक्।
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Pratyāhāras

1.			a	i	u	Ṇ
2.				ṛ	ḷ	K
3.				e	o	Ṅ
4.				ai	au	C
5.		h	y	v	r	Ṭ

Pratyāhāras

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2.				ṛ	ḷ	K
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iK

Pratyāhāras

1.			a	i	u	Ṇ
2.				ṛ	ḷ	K
3.				e	o	Ṅ
4.				ai	au	C
5.		h	y	v	r	Ṭ

iK= ⟨i, u, ṛ, ḷ⟩

Analysis of iko yaṇaci: [iK] → [yṆ]/_ [aC]

1.			a	i	u	Ṇ
2.				ṛ	ḷ	K
3.				e	o	Ṅ
4.				ai	au	C
5.		h	y	v	r	Ṭ
6.					l	Ṇ

- [iK] → [yṆ]/_ [aC]
- ⟨i, u, ṛ, ḷ⟩ → ⟨y, v, r, l⟩/_ ⟨a, i, u, ṛ, ḷ, e, o, ai, au⟩

Analysis of iko yaṇaci: [iK] → [yṆ]/__[aC]

1.			a	i	u	Ṇ
2.				ṛ	ḷ	K
3.				e	o	Ṅ
4.				ai	au	C
5.	h		y	v	r	Ṭ
6.					l	Ṇ

- [iK] → [yṆ]/__[aC]
- ⟨i, u, ṛ, ḷ⟩ → ⟨y, v, r, l⟩/__⟨a, i, u, ṛ, ḷ, e, o, ai, au⟩

General problem of S-sortability

Given a set of classes, order the elements of the classes (without duplications) in a linear order (in a list) such that each single class forms a continuous interval with respect to that order.

- The target orders are called S-orders
- A set of classes is S-sortable if it has an S-order

General problem of *Śivasūtra*-alphabets (S-alphabets)

Given a set of classes, find an S-order of the elements of the classes. Interrupt this list by markers such that each single class can be denoted by a sound-marker-pair (*pratyāhāra*).

Note that every S-order becomes a *Śivasūtra*-alphabet (S-alphabet) by adding a marker behind each element.

Given the set of classes $\{\{a, b\}, \{a, b, c\}, \{a, b, c, d\}\}$, the order $a\ b\ c\ d$ is one of its S-orders and $a\ M_1\ b\ M_2\ c\ M_3\ d\ M_4$ is one of its S-alphabets.

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Some more Examples

S-sortable example

The set of classes:
$\{\{d, e\}, \{a, b\}, \{b, c, d, f, g, h, i\}, \{f, i\}, \{c, d, e, f, g, h, i\}, \{g, h\}\}$ is S-sortable;
one of its S-orders is
$a\,b\,c\,g\,h\,f\,i\,d\,e$

non-S-sortable example

The set of classes:
$\{\{a, b\}, \{b, c\}, \{a, c\}\}$ is not S-sortable.

non-S-sortable example

The set of classes:
$\{\{d, e\}, \{a, b\}, \{b, c, d\}, \{b, c, d, f\}\}$ is not S-sortable.

Some more Examples

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$a\,b\,c\,d\,e$ or $e\,d\,c\,b\,a$

Some more Examples

S-sortable example

The set of classes:
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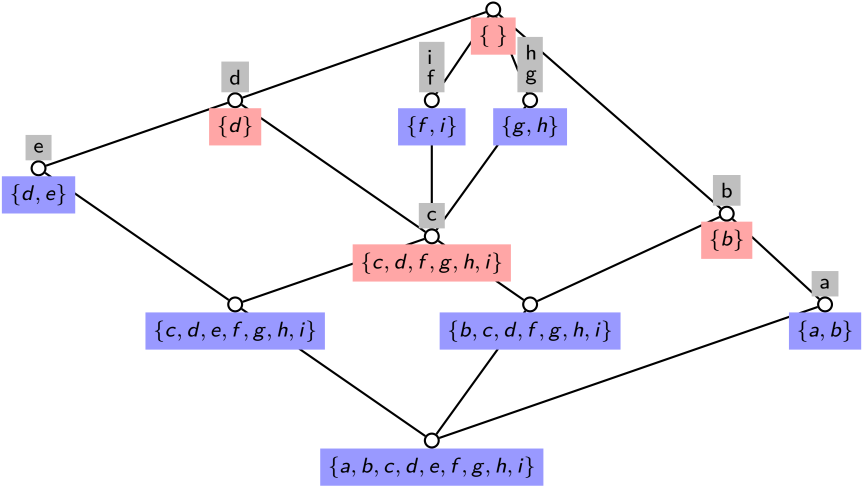
Visualize relations

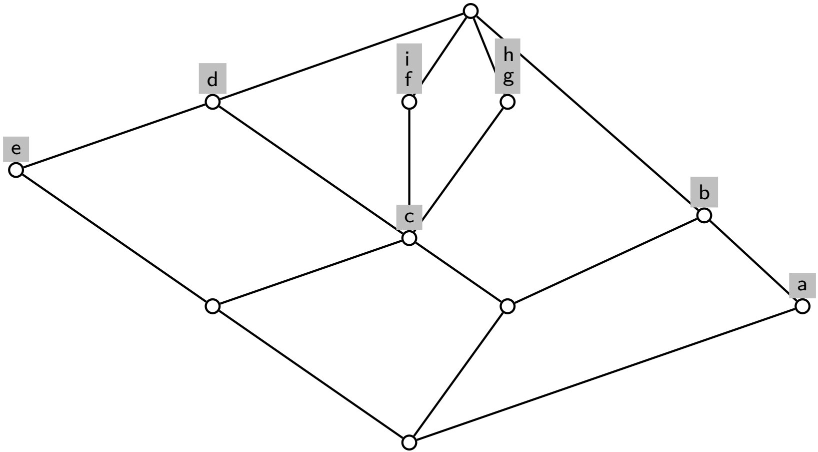
$$\{\{d,e\},\{a,b\},\{b,c,d,f,g,h,i\},\{f,i\},\{c,d,e,f,g,h,i\},\{g,h\}\}$$



concept lattice

Visualize relations

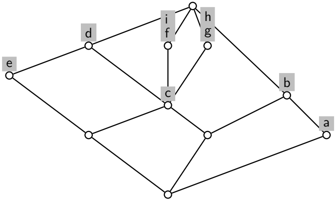
$\{\{d,e\},\{a,b\},\{b,c,d,f,g,h,i\},\{f,i\},\{c,d,e,f,g,h,i\},\{g,h\}\}$



concept lattice

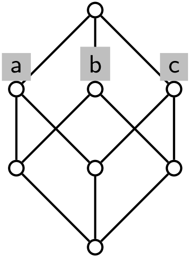
Visualize relations

$\{\{d, e\}, \{a, b\}, \{b, c, d, f, g, h, i\},$
$\{f, i\}, \{c, d, e, f, g, h, i\}, \{g, h\}\}$



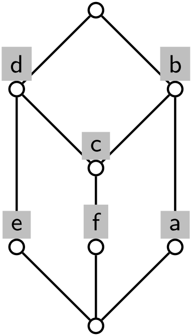
S-sortable

$\{\{a, b\}, \{b, c\}, \{a, c\}\}$



not S-sortable

$\{\{d, e\}, \{a, b\}, \{b, c, d\},$
$\{b, c, d, f\}\}$



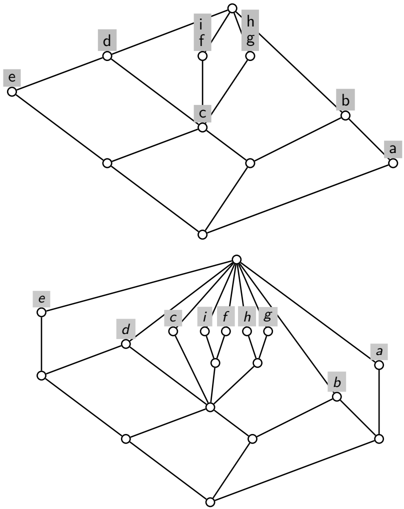
not S-sortable

Main theorem of S-sortability

A set of classes is S-sortable without duplications if one of the following equivalent statements is true:

1. Its concept lattice is Hasse-planar and for any element *a* there is a node labeled *a* in the S-graph.
2. The concept lattice of the enlarged set of classes is Hasse-planar.
3. The Ferrers-graph of the enlarged set of classes is bipartite.

S-sortable example



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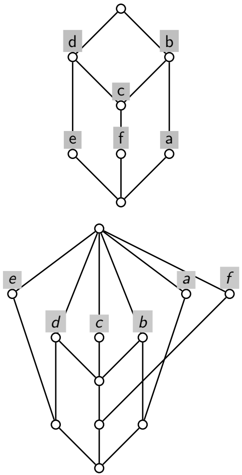
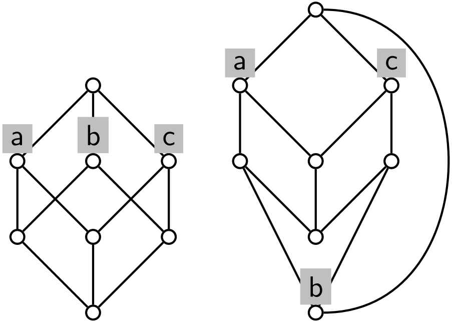
not S-sortable example



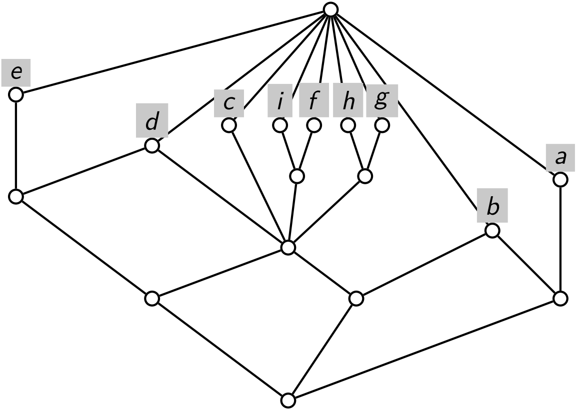
Hasse-planarity

$$\{\{a, b\}, \{a, c\}, \{b, c\}\}$$



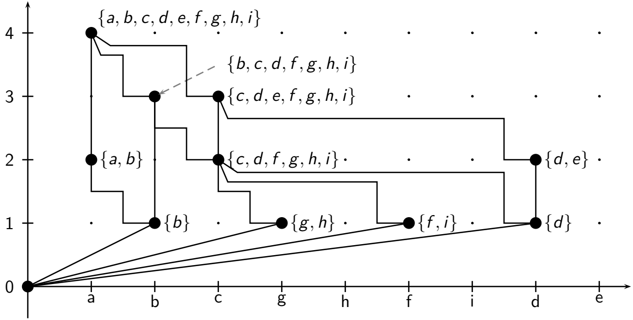
planar, but not Hasse-planar

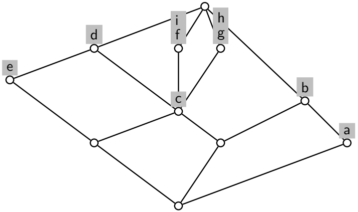
2nd condition: Hasse-planar ⇒ S-sortable

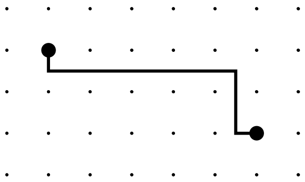


$\{\{d, e\}, \{a, b\}, \{b, c, d, f, g, h, i\}, \{f, i\}, \{c, d, e, f, g, h, i\}, \{g, h\}\}$

2nd condition: S-sortable ⇒ Hasse-planar







2nd condition: evaluation

- It is of no help in the construction of S-alphabets with minimal number of markers.
- The planarity of a graph is difficult to check.

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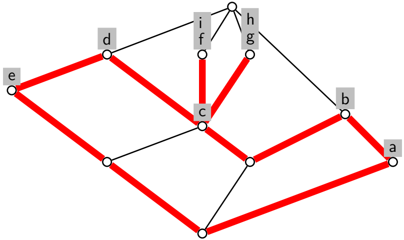
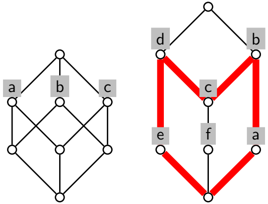
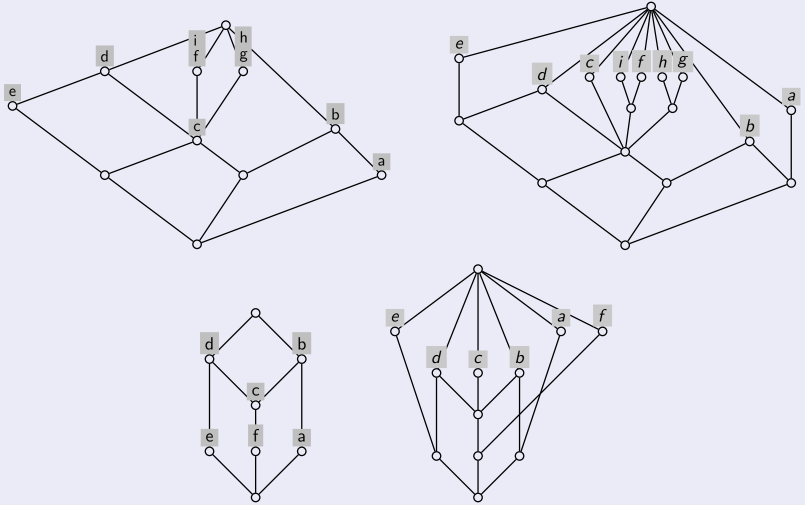
S-sortable example



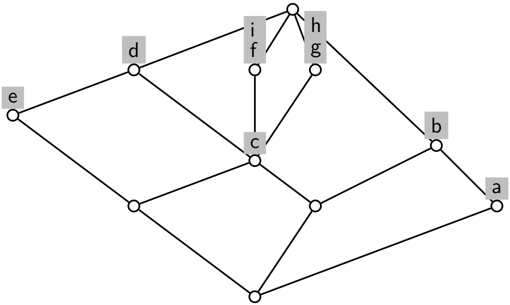
non S-sortable examples



1st condition ⇔ 2nd condition



S-alphabets with a minimal number of markers

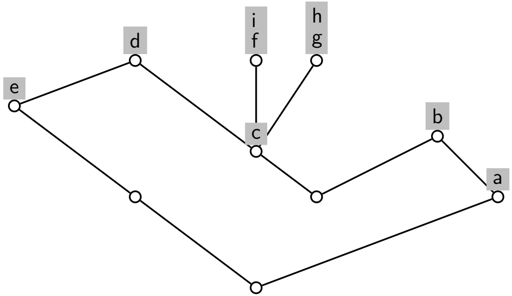


procedure

Start with the empty sequence and choose a walk through the S-graph:

- While moving upwards do nothing.
- While moving downwards along an edge add a new marker to the sequence unless its last element is already a marker.
- If a labeled node is reached, add the labels in arbitrary order to the sequence, unless it has been added before.

S-alphabets with a minimal number of markers



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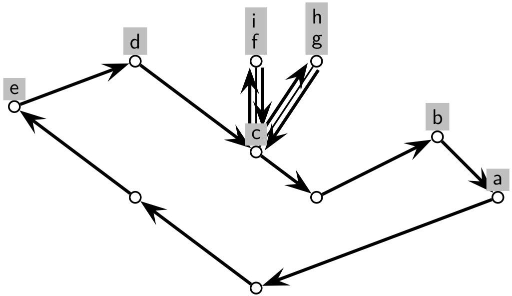
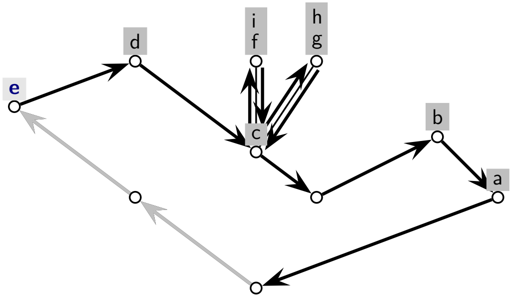


procedure

Start with the empty sequence and choose a walk through the S-graph:

- While moving upwards do nothing.
- While moving downwards along an edge add a new marker to the sequence unless its last element is already a marker.
- If a sound is reached, add the sound to the sequence, unless it has been added before.

S-alphabets with a minimal number of markers



procedure

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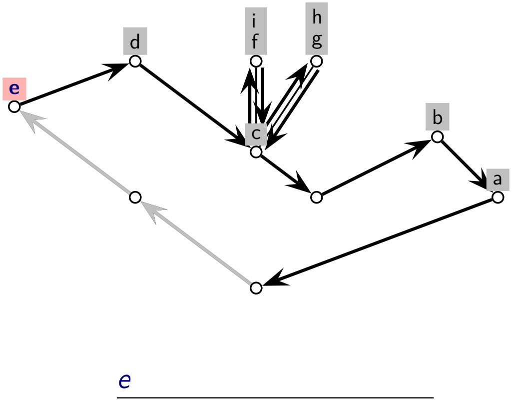
- If a sound is reached, add the sound to the sequence, unless it has been added before.

S-alphabets with a minimal number of markers



procedure

Start with the empty sequence and choose a walk through the S-graph:

- While moving upwards do nothing.
- While moving downwards along an edge add a new marker to the sequence unless its last element is already a marker.
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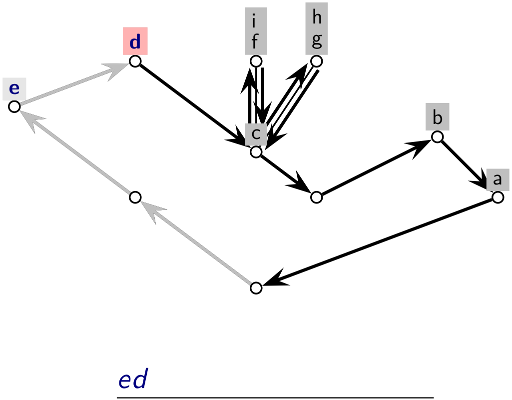

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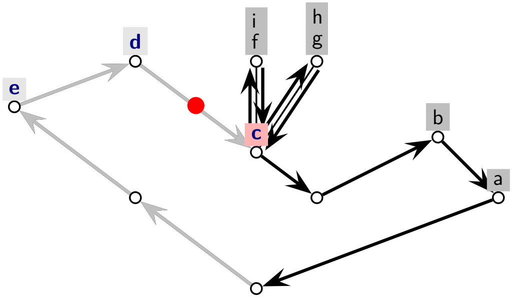

$ed\,M_1\,c$

procedure

Start with the empty sequence and choose a walk through the S-graph:

- While moving upwards do nothing.
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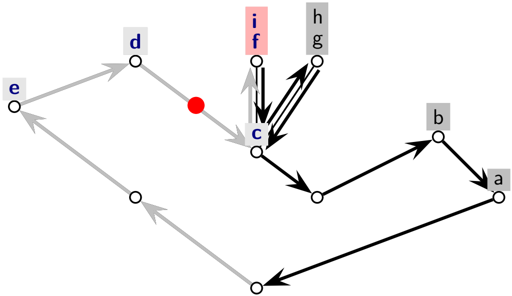
S-alphabets with a minimal number of markers



$ed\,M_1\,cfi$

procedure

Start with the empty sequence and choose a walk through the S-graph:

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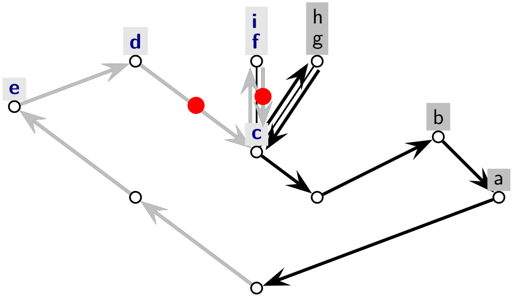
S-alphabets with a minimal number of markers



$ed\,M_1\,cfi\,M_2$

procedure

Start with the empty sequence and choose a walk through the S-graph:

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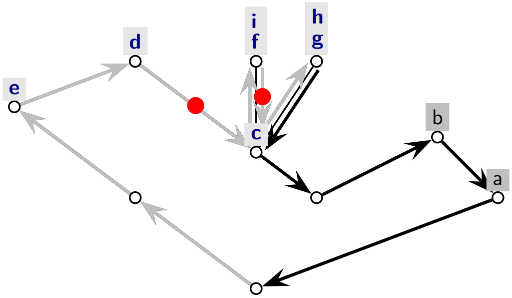
S-alphabets with a minimal number of markers



$ed\,M_1\,cfi\,M_2\,gh$

procedure

Start with the empty sequence and choose a walk through the S-graph:

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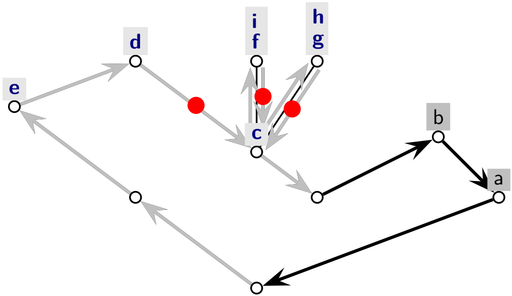

$ed\,M_1\,cfi\,M_2\,gh\,M_3$

procedure

Start with the empty sequence and choose a walk through the S-graph:

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- While moving downwards along an edge add a new marker to the sequence unless its last element is already a marker.
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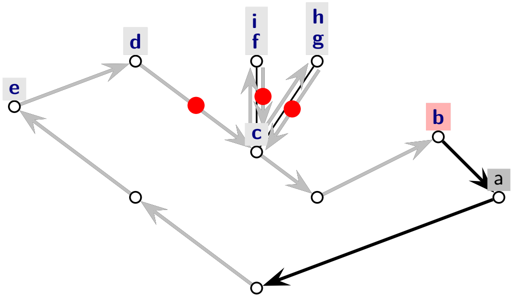
S-alphabets with a minimal number of markers



$ed\,M_1\,cfi\,M_2\,gh\,M_3\,b$

procedure

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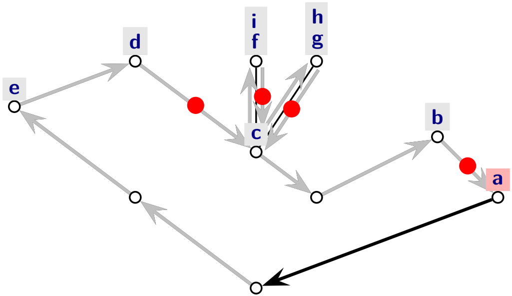
S-alphabets with a minimal number of markers



ed M_1 *cfi* M_2 *gh* M_3 *b* M_4 *a*

procedure

Start with the empty sequence and choose a walk through the S-graph:

- While moving upwards do nothing.
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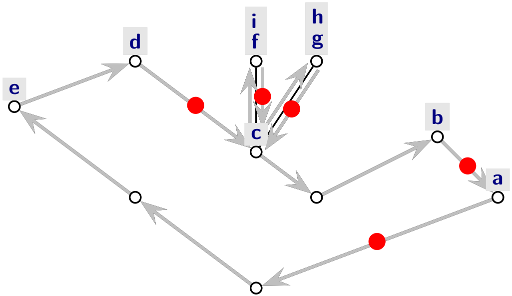
S-alphabets with a minimal number of markers



$ed\,M_1\,c\,fi\,M_2\,gh\,M_3\,b\,M_4\,a\,M_5$

procedure

Start with the empty sequence and choose a walk through the S-graph:

- While moving upwards do nothing.
- While moving downwards along an edge add a new marker to the sequence unless its last element is already a marker.
- If a sound is reached, add the sound to the sequence, unless it has been added before.

1st condition: evaluation

+ Allows the construction of S-alphabets with minimal number of markers.
− The planarity of a graph is difficult to check.

Main theorem of S-sortability

A set of classes is S-sortable without duplications if one of the following equivalent statements is true:

1. Its concept lattice is Hasse-planar and for any element *a* there is a node labeled *a* in the S-graph.
2. The concept lattice of the enlarged set of classes is Hasse-planar.
3. The Ferrers-graph of the enlarged set of classes is bipartite.

- The Ferrers-graph can be computed directly from the set of classes.
- Its bipartity can be checked algorithmically.

skip

3rd condition: terminology & proof

Theorem (Zschalig 2007)

The concept lattice of a formal context is Hasse-planar if and only if its Ferrers-graph is bipartite.

	a	b	c	d	e	f
0				×	×	
1		×	×	×		
2	[×]	×				
3		×	[×]			×

3rd condition: terminology & proof

Theorem (Zschalig 2007)

The concept lattice of a formal context is Hasse-planar if and only if its Ferrers-graph is bipartite.

	a	b	c	d	e	f
0	●	●	●	×	×	●
1	●	×	×	×	●	●
2	[×]	×	●	●	●	●
3	●	×	[×]	●	●	×

3rd condition: terminology & proof

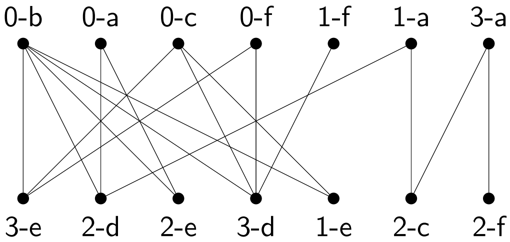
Theorem (Zschalig 2007)

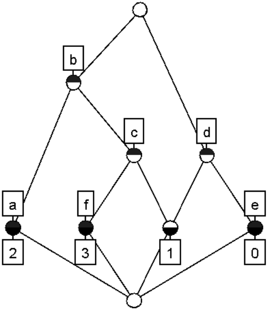
The concept lattice of a formal context is Hasse-planar if and only if its Ferrers-graph is bipartite.

	a	b	c	d	e	f
0	●	●	●	×	×	●
1	●	×	×	×	●	●
2	×	×	●	●	●	●
3	●	×	×	●	●	×

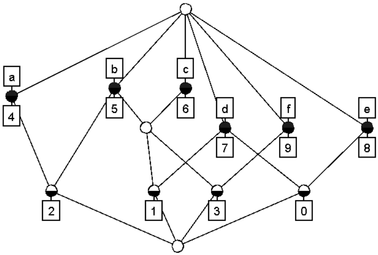
3rd condition: example

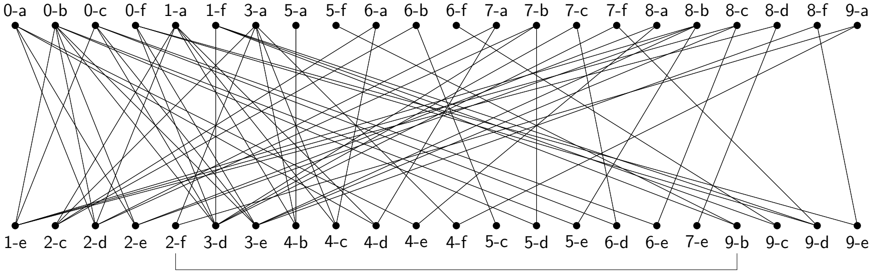
	a	b	c	d	e	f
0				×	×	
1		×	×	×		
2	×	×				
3		×	×			×





3rd condition: example





3rd condition: evaluation

- – It is of no help in the construction of S-alphabets with minimal number of markers.
- \+ It can be checked easily by an algorithm.

Getting back to Pāṇini's problem

a·i·uṇ| ṛ·ḷk| e·oṅ| ai·auc| hayavaraṭ|
laṇ| ñamaṅaṇanam| jhabhañ| ghaḍhadhaṣ| jabagaḍadaś|
khaphachaṭhathacaṭatav| kapay| śaṣasar| hal|

Q: Are the *Śivasūtras* minimal (with respect to length)?

What does minimal mean?

a·i·uṇ| *ṛ·ḷk*| *e·oṅ*| *ai·auc*| *hayavaraṭ*|
laṇ| *ñamaṅaṇanam*| *jhabhañ*| *ghaḍhadhaṣ*| *jabagaḍadaś*|
khaphachaṭhathacaṭatav| *kapay*| *śaṣasar*| *hal*|

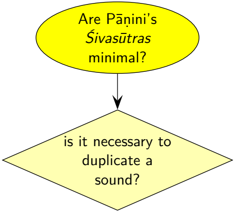
The *Śivasūtras* are **not minimal** if it is possible to rearrange the Sanskrit sounds in a new list with markers such that

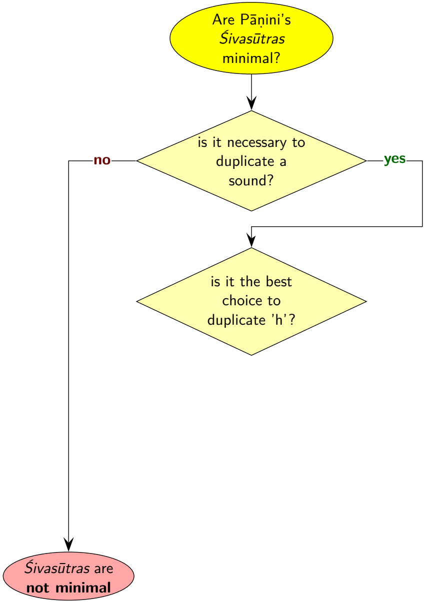
1. each *pratyāhāra* forms an interval ending before a marker,
2. no sound occurs twice

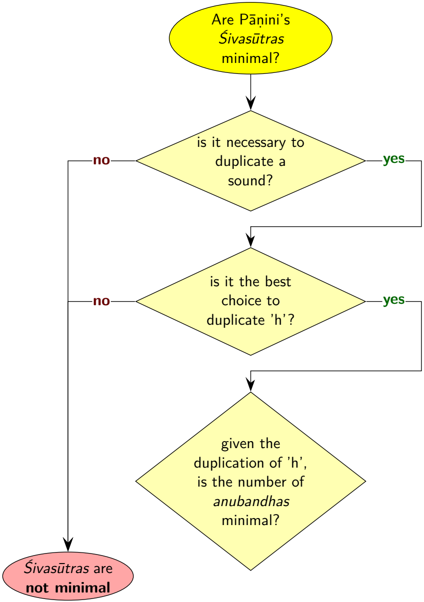
or one sound occurs twice but less markers are needed.

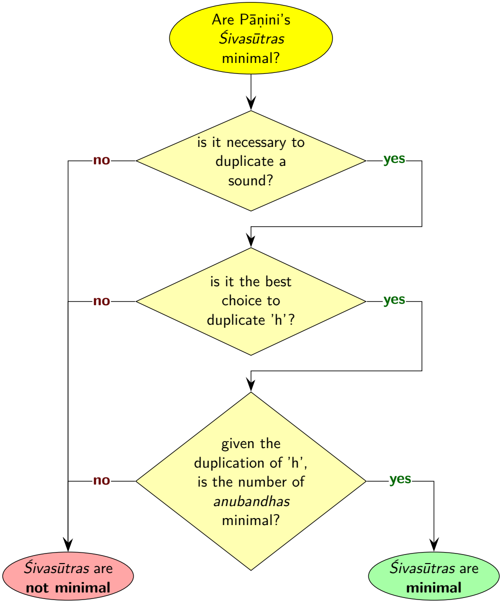
⇒ duplicating a sound is worse than adding markers

Are Pāṇini's *Śivasūtras* minimal?


Are Pāṇini's *Śivasūtras* minimal?
is it necessary to duplicate a sound?


Are Pāṇini's *Śivasūtras* minimal?
is it necessary to duplicate a sound?
no
yes
is it the best choice to duplicate 'h'?
Śivasūtras are **not minimal**


Are Pāṇini's *Śivasūtras* minimal?
is it necessary to duplicate a sound?
no
yes
is it the best choice to duplicate 'h'?
no
yes
given the duplication of 'h', is the number of *anubandhas* minimal?
Śivasūtras are **not minimal**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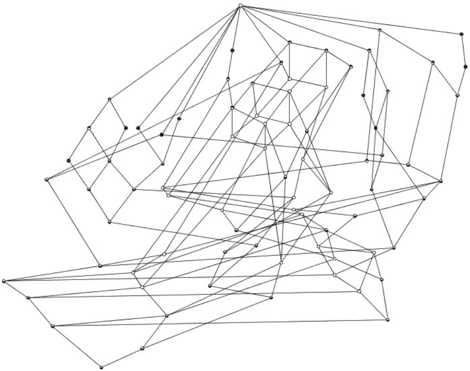
Śivasūtras are **not minimal**
Śivasūtras are **minimal**

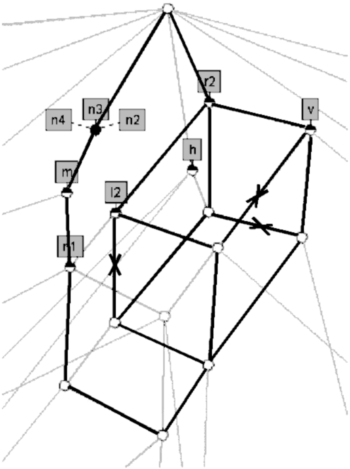
Is it necessary to duplicate a sound?

Main theorem on S-sortability (part 1a)

If a set of classes is S-sortable, then its concept lattice is Hasse-planar.

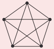


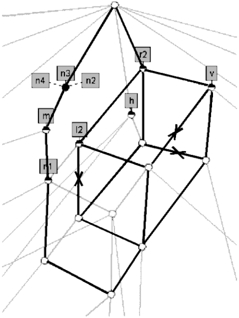
concept lattice of Pāṇini's *pratyāhāras*



Is it necessary to duplicate a sound?

Criterion of Kuratowski

A graph which has the graph  as a minor is not planar.



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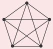
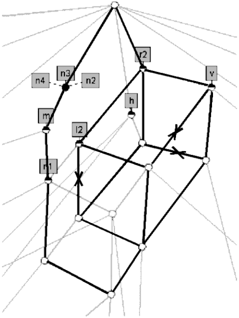
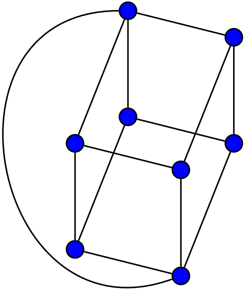
A graph which has the graph  as a minor is not planar.





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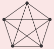
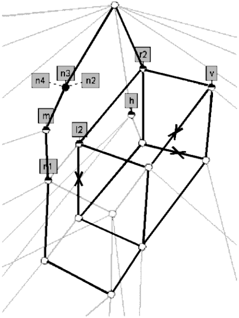
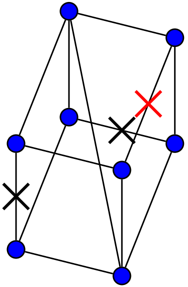
Criterion of Kuratowski

A graph which has the graph  as a minor is not planar.





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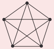
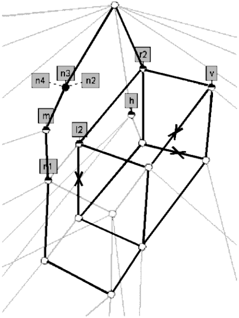
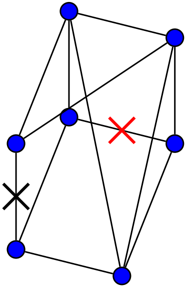
A graph which has the graph  as a minor is not planar.

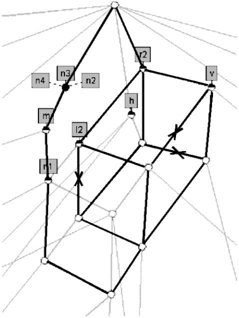


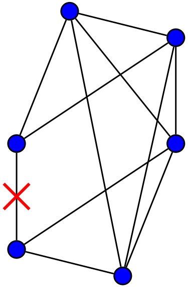


Is it necessary to duplicate a sound?

Criterion of Kuratowski

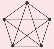
A graph which has the graph  as a minor is not planar.

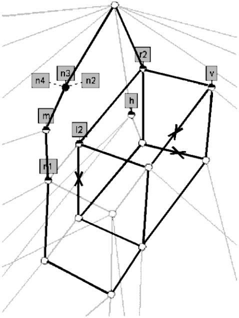


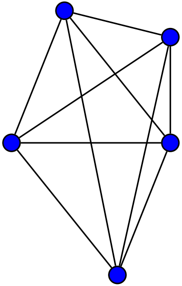


Is it necessary to duplicate a sound?

Criterion of Kuratowski

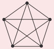
A graph which has the graph  as a minor is not planar.

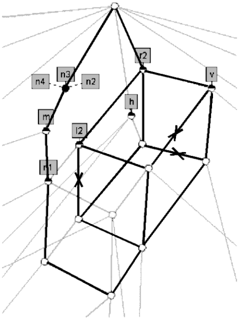


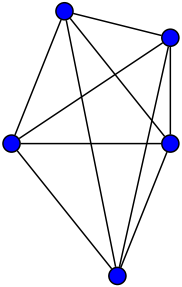


Is it necessary to duplicate a sound?

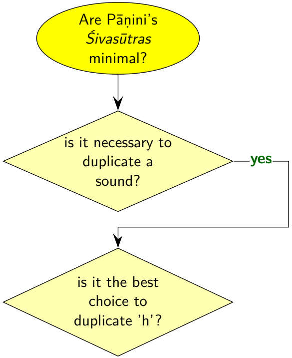
Criterion of Kuratowski

A graph which has the graph  as a minor is not planar.



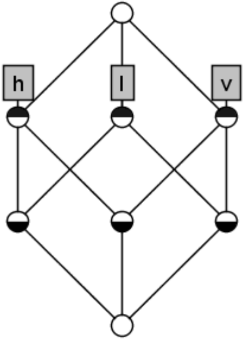


There is no S-alphabet for the set of classes given by Pāṇini's *pratyāhāras* without duplicated elements!

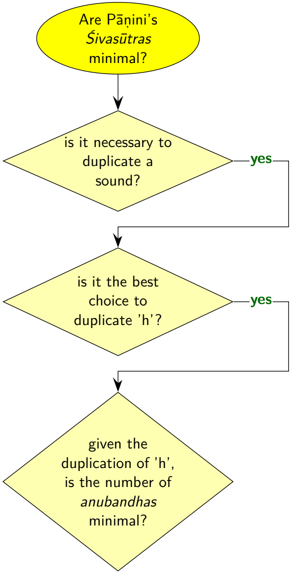

Are Pāṇini's *Śivasūtras* minimal?
is it necessary to duplicate a sound?
yes
is it the best choice to duplicate 'h'?

h and the independent triples

	h	l	v
$\{h, l\}$	×	×	
$\{h, v\}$	×		×
$\{v, l\}$		×	×



Altogether there exists 249 independent triples.
h is included in all of them.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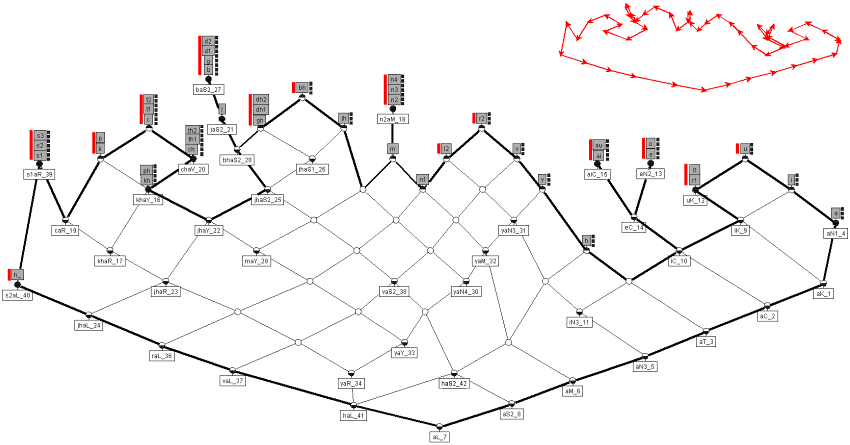
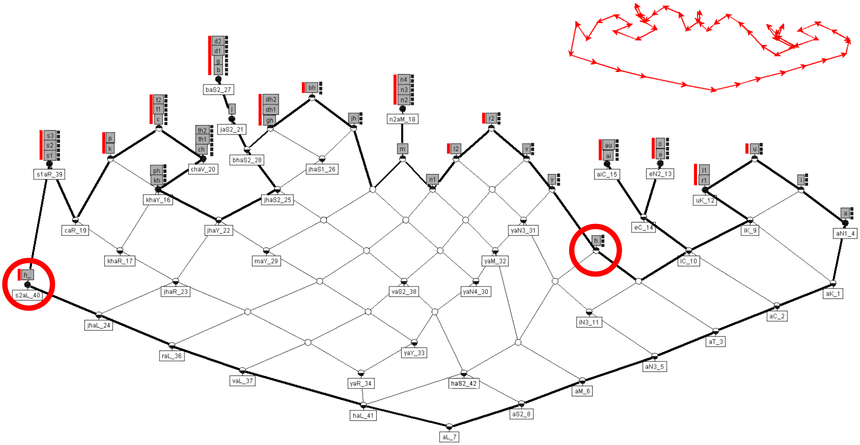
yes
is it the best choice to duplicate 'h'?
yes
given the duplication of 'h', is the number of *anubandhas* minimal?

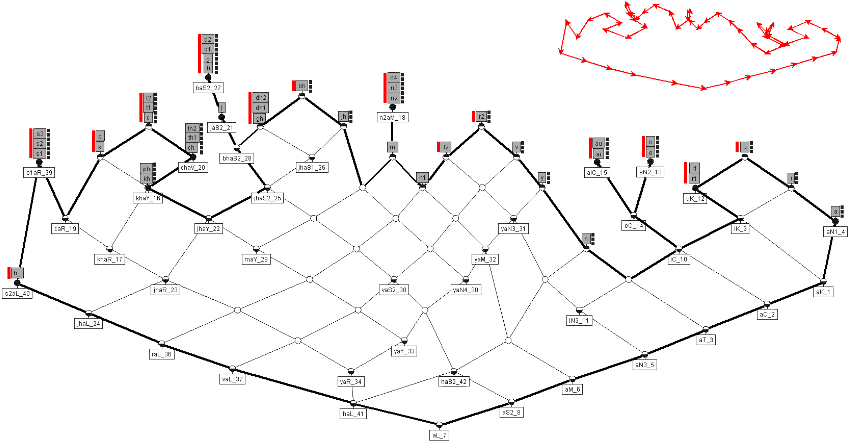
Concept lattice of Pāṇini's *pratyāhāras* with duplicated *h*



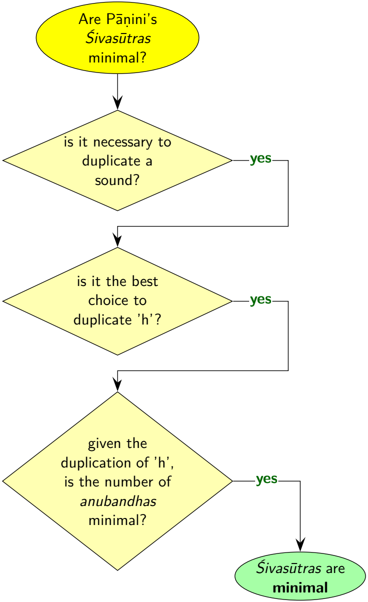
Concept lattice of Pāṇini's *pratyāhāras* with duplicated *h*



Concept lattice of Pāṇini's *pratyāhāras* with duplicated *h*



With the *Śivasūtras* Pāṇini has chosen one out of nearly 12 million minimal S-alphabets!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is it necessary to duplicate a sound?
yes
is it the best choice to duplicate 'h'?
yes
given the duplication of 'h', is the number of *anubandhas* minimal?
yes
Śivasūtras are **minimal**

Open problems

The story is much more intricate

- We have neither shown that Pāṇini's technique for the representation of sound classes is optimal
- nor that he has used his technique in an optimal way.
  - not all sound classes are denoted by *pratyāhāras*
  - rules overgeneralize
  - *sūtra* 1.3.10: *yathāsaṃkhyamanudeśaḥ samānām*

Open problems

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- nor that he has used his technique in an optimal way.
  - not all sound classes are denoted by *pratyāhāras*
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  - *sūtra* 1.3.10: *yathāsaṃkhyamanudeśaḥ samānām*

$$\langle a, i, u, M_1, \{ṛ, ḷ\}_1, M_2, \{\langle\{e, o\}_2, M_3\rangle, \langle\{ai, au\}_3, M_4\rangle\}_4,$$
$$h, y, v, r, M_5, l, M_6, ñ, m, \{ṅ, ṇ, n\}_5, M_7, jh, bh, M_8,$$
$$\{gh, ḍh, dh\}_6, M_9, j, \{b, g, ḍ, d\}_7, M_{10}, \{kh, ph\}_8, \{ch, ṭh, th\}_9,$$
$$\{c, ṭ, t\}_{10}, M_{11}, \{k, p\}_{11}, M_{12}, \{ś, ṣ, s\}_{12}, M_{13}, h, M_{14}\rangle$$

$$\underset{\{\}_1}{2!} \times \underset{\{\}_2}{2!} \times \underset{\{\}_3}{2!} \times \underset{\{\}_4}{2!} \times \underset{\{\}_5}{3!} \times \underset{\{\}_6}{3!} \times \underset{\{\}_7}{4!} \times \underset{\{\}_8}{2!} \times \underset{\{\}_9}{3!} \times \underset{\{\}_{10}}{3!} \times \underset{\{\}_{11}}{2!} \times \underset{\{\}_{12}}{3!}$$
$$= 2 \times 2 \times 2 \times 2 \times 6 \times 6 \times 24 \times 2 \times 6 \times 6 \times 2 \times 6 = 11\,943\,936$$

Some numbers

- Pāṇini denotes 42 sound classes by *pratyāhāras*.
- The *Śivasūtras* allow the construction of 281 *pratyāhāras*.
- $2^{42} - 43$ ($> 2 \cdot 10^{12}$) possible sound classes.
- 11 (resp. 10, if unmarked classes are permitted) binary features are necessary to denote Pāṇini's *pratyāhāras* ($\Rightarrow 2^{11} = 2048$, resp. $2^{10} = 1024$ classes can be constructed).
- Pāṇini has chosen 1 out of 11.943.936 minimal S-alphabets
- The 42 sounds can be ordered in nearly 43! ($> 6 \cdot 10^{52}$) lists in which *h* occurs twice.

Origin of Pictures

- libraries (left): `http://www.meduniwien.ac.at/medizinischepsychologie/bibliothek.htm`
- libraries (middle): `http://www.math-nat.de/aktuelles/allgemein.htm`
- libraries (right): `http://www.geschichte.mpg.de/deutsch/bibliothek.html`
- warehouses: `http://www.metrogroup.de/servlet/PB/menu/1114920_l1/index.html`
- stores: `http://www.einkaufsparadies-schmidt.de/01bilder01/`