On the minimality of Pāṇini's *Śivasūtras*

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अइउण्। ऋऌक्। एओङ्। ऐऔच्। हयवरट्।
लण्। ञमङणनम्। झभञ्। घढधष्। जबगडदश्।
खफछठथचटतव्। कपय्। शषसर्। हल्।

Pāṇini's *Śivasūtras*

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a·i·uṇ | *ṛ·ḷk* | *e·oṅ* | *ai·auc* | *hayavaraṭ* |
laṇ | *ñamaṅaṇanam* | *jhabhañ* | *ghaḍhadhaṣ* | *jabagaḍadaś* |
khaphachaṭhathacaṭatav | *kapay* | *śaṣasar* | *hal* |

Pāṇini's *Śivasūtras* in tabular form

1.	a	i	u			Ṇ
2.				ṛ	ḷ	K
3.		e	o			Ṅ
4.		ai	au			C
5.	h	y	v	r		Ṭ
6.					l	Ṇ
7.	ñ	m	ṅ	ṇ	n	M
8.	jh	bh				Ñ
9.			gh	ḍh	dh	Ṣ
10.	j	b	g	ḍ	d	Ś
11.	kh	ph	ch	ṭh	th	
			c	ṭ	t	V
12.	k	p				Y
13.		ś	ṣ	s		R
14.	h					L

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hayavaraṭ| laṇ|

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anubandha

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Pratyāhāras

1.	a	i	u			Ṇ
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4.		ai	au			C
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- a *pratyāhāra* is a pair of a sound and an *anubandha*
- it denotes the continuous sequence of sounds in the interval between the sound and the *anubandha*

Pratyāhāras

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iK= ⟨i, u, ṛ, ḷ⟩

- a *pratyāhāra* is a pair of a sound and an *anubandha*
- it denotes the continuous sequence of sounds in the interval between the sound and the *anubandha*

Phonological Rules

modern notation

A is replaced by B if preceded by C and succeeded by D.

$$A \rightarrow B / _{C\_\_D}$$

example: final devoicing

$$\begin{bmatrix} + & \text{consonantal} \\ - & \text{nasal} \\ + & \text{voiced} \end{bmatrix} \rightarrow \begin{bmatrix} + & \text{consonantal} \\ - & \text{nasal} \\ - & \text{voiced} \end{bmatrix} / \_\_\sharp$$

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Pāṇini's linear Coding

$$A + \text{genitive}, B + \text{nominative}, C + \text{ablative}, D + \text{locative}.$$

example

- *sūtra* 6.1.77: *iko yaṇaci* (इको यणचि)
- analysis: $[\text{ik}]_{\text{gen}}[\text{yaṇ}]_{\text{nom}}[\text{ac}]_{\text{loc}}$
- modern notation: $[\text{iK}] \rightarrow [\text{yṆ}]/\_\_[\text{aC}]$

Phonological Rules

modern notation

A is replaced by B if preceded by C and succeeded by D.

$$A \to B/_{C\_\_D}$$

Pāṇini's linear Coding

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example

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Analysis of iko yaṇaci: [iK] → [yṆ]/_ [aC]

1.	a	i	u			Ṇ
2.				ṛ	ḷ	K
3.		e	o			Ṅ
4.		ai	au			C
5.	h	y	v	r		Ṭ
6.					l	Ṇ

- [iK] → [yṆ]/_ [aC]
- ⟨i, u, ṛ, ḷ⟩ → ⟨y, v, r, l⟩/_ ⟨a, i, u, ṛ, ḷ, e, o, ai, au⟩

Analysis of iko yaṇaci: [iK] → [yṆ]/_ [aC]

1.	a	i	u			Ṇ
2.				ṛ	ḷ	K
3.		e	o			Ṅ
4.		ai	au			C
5.	h	y	v	r		Ṭ
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- [iK] → [yṆ]/_ [aC]
- ⟨i, u, ṛ, ḷ⟩ → ⟨y, v, r, l⟩/_ ⟨a, i, u, ṛ, ḷ, e, o, ai, au⟩

Analysis of iko yaṇaci: [iK] → [yṆ]/__[aC]

1.	a	i	u			Ṇ
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3.		e	o			Ṅ
4.		ai	au			C
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6.					l	Ṇ

- [iK] → [yṆ]/__[aC]
- ⟨i, u, ṛ, ḷ⟩ → ⟨y, v, r, l⟩/__⟨a, i, u, ṛ, ḷ, e, o, ai, au⟩

Possible minimality criteria

total list:

a i u Ṇ ṛ ḷ K e o Ṅ ai au C h y v r Ṭ l Ṇ ñ m ṅ ṇ n M jh bh Ñ gh ḍh dh Ṣ j b g ḍ d Ś kh ph ch ṭh th c ṭ t V k p Y ś ṣ s R h L

1. total list is of minimal length;
2. sound list is of minimal length;
3. *anubandha* list is of minimal length;
4. total list is as short as possible while the *anubandha* list is minimal;
5. total list is as short as possible while the sound list is minimal;

Possible minimality criteria

sound list:

a i u   ṛ ḷ   e o   ai au   h y v r   l   ñ m ṅ ṇ n   jh bh   gh ḍh dh
j b g ḍ d   kh ph ch ṭh th c ṭ t   k p   ś ṣ s   h

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Possible minimality criteria

anubandha list:

Ṇ K Ṅ Ś C Ṭ Ṇ V Y M R Ñ L Ṣ

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5. total list is as short as possible while the sound list is minimal;

⇒ duplicating sounds is worse than adding *anubandhas*

Principle of economy

Staal 1962

Another general principle is also implicitly used by Pāṇini. This is the famous economy criterion [. . .] In accordance with this principle each linguistic rule should be given in the shortest possible form, whereas the number of metalinguistic symbols should be reduced as far as possible.

$\Rightarrow$ 5. criterion of minimality: total list is as short as possible while the sound list is minimal

Example: semi-formal argument

Kiparsky 1991

The reasoning from economy goes like this. To be grouped together in a pratyāhāra, sounds must make up a continuous segment of the list. Economy requires making the list as short as possible, which means avoiding repetitions of sounds, and using as few markers as possible. Consequently, if class A properly includes class B, the elements shared with B should be listed last in A; the marker that follows can then be used to form pratyāhāras for both A and B. In this way the economy principle, by selecting the shortest grammar, determines both the ordering of sounds and the placement of markers among them.

Example: semi-formal argument

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Example: semi-formal argument

***Śivasūtras*:**

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$aK = \{a, i, u, ṛ, ḷ\}$, $iK = \{i, u, ṛ, ḷ\}$ and $uK = \{u, ṛ, ḷ\} \Rightarrow a < i < u < ṛ,ḷ$

but:

$jhL =$
$\{h, s, ṣ, ś, p, k, t, ṭ, c, th, ṭh, ch, ph, kh, d, ḍ, g, b, j, dh, ḍh, gh, bh, jh\}$

$jhR =$
$\{s, ṣ, ś, p, k, t, ṭ, c, th, ṭh, ch, ph, kh, d, ḍ, g, b, j, dh, ḍh, gh, bh, jh\}$

$jhY = \{p, k, t, ṭ, c, th, ṭh, ch, ph, kh, d, ḍ, g, b, j, dh, ḍh, gh, bh, jh\}$

$jhŚ = \{d, ḍ, g, b, j, dh, ḍh, gh, bh, jh\}$ and

$jhṢ = \{dh, ḍh, gh, bh, jh\}$

$\Rightarrow h < s, ṣ, ś < p, k, t, ṭ, c, th, ṭh, ch, ph, kh, d < ḍ, g, b, j <$
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Example: semi-formal argument

Śivasūtras:

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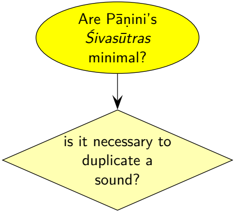
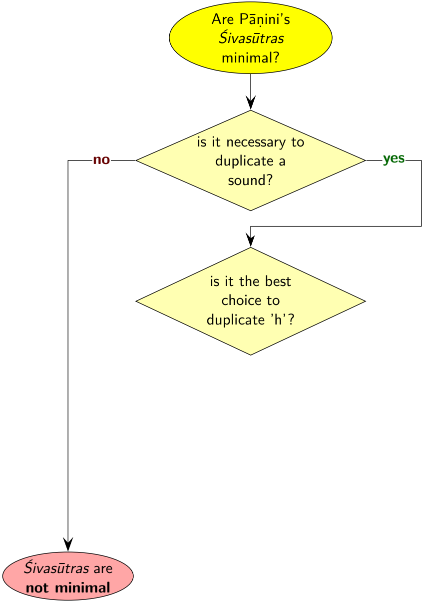
$jhŚ = \{d, ḍ, g, b, j, dh, ḍh, gh, bh, jh\}$ and

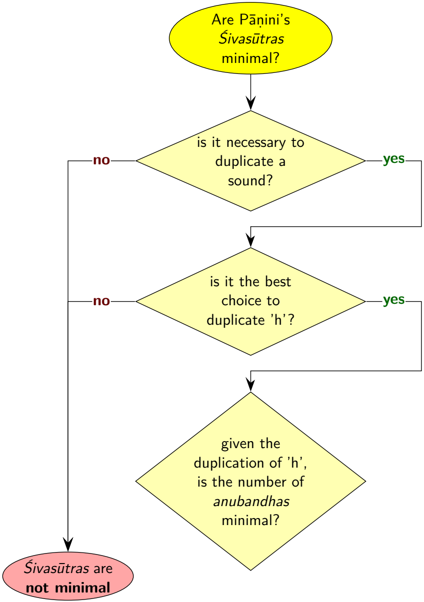
$jhṢ = \{dh, ḍh, gh, bh, jh\}$

$\Rightarrow h < s, ṣ, ś < p, k, t, ṭ, c, th, ṭh, ch, ph, kh, d < ḍ, g, b, j <$
$dh, ḍh, gh, bh, jh$

Are Pāṇini's *Śivasūtras* minimal?


Are Pāṇini's *Śivasūtras* minimal?
is it necessary to duplicate a sound?


Are Pāṇini's *Śivasūtras* minimal?
is it necessary to duplicate a sound?
no
yes
is it the best choice to duplicate 'h'?
Śivasūtras are **not minimal**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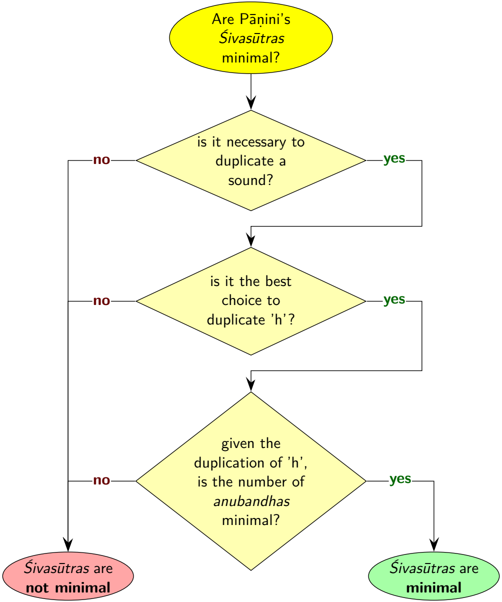
Śivasūtras are **not minimal**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given the duplication of 'h', is the number of *anubandhas* minimal?
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Śivasūtras are **not minimal**
Śivasūtras are **minimal**

Terminology: S-encodability

set of classes $(\mathcal{A}, \Phi)$: $\mathcal{A} = \{a, b, c, d, e, f, g, h, i\}$

$$\Phi = \{\{d, e\}, \{a, b\}, \{b, c, d, f, g, h, i\}, \{f, i\}, \{c, d, e, f, g, h, i\}, \{g, h\}\}$$

S-Alphabet $(\mathcal{A}, \Sigma, <)$ of $(\mathcal{A}, \Phi)$: $a\ b\ M_1\ c\ g\ h\ M_2\ f\ i\ M_3\ d\ M_4\ e\ M_5$

set of sounds

total order on $\mathcal{A} \cup \Sigma$

set of markers

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set of sounds

set of markers

total order on $\mathcal{A} \cup \Sigma$

$\Rightarrow$ $(\mathcal{A}, \Phi)$ is S-encodable without duplications

Terminology: S-encodability

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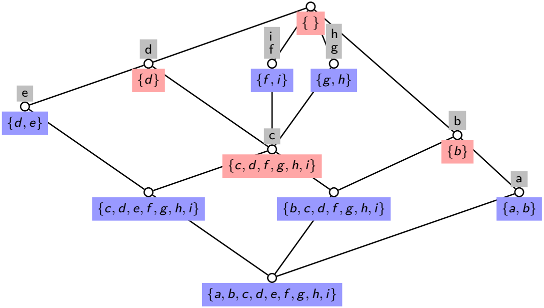
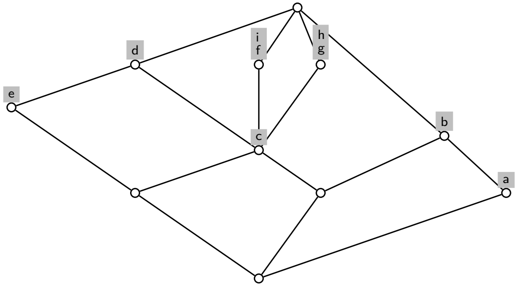


concept lattice of $(\mathcal{A}, \Phi)$

Terminology: S-encodability

set of classes $(\mathcal{A}, \Phi)$: $\mathcal{A} = \{a, b, c, d, e, f, g, h, i\}$

$$\Phi = \{\{d, e\}, \{a, b\}, \{b, c, d, f, g, h, i\}, \{f, i\}, \{c, d, e, f, g, h, i\}, \{g, h\}\}$$



concept lattice of $(\mathcal{A}, \Phi)$

Generalized task

- Given the sound classes which are denoted by *pratyāhāras* in Pāṇini's grammar, do the *Śivasūtras* fulfill the 5th minimality criterion?
- Does an S-alphabet which fulfills the 5th minimality criterion exists for any arbitrary set of classes and how can it be constructed?

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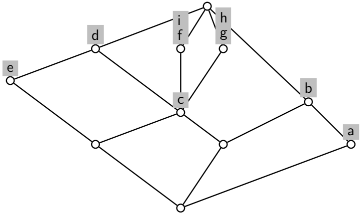
S-encodability ⇒ planarity

Main theorem on S-encodability (part 1)

The concept lattice of $(\mathcal{A}, \Phi)$ is planar if $(\mathcal{A}, \Phi)$ is S-encodable without duplications



S-encodability $\Rightarrow$ planarity

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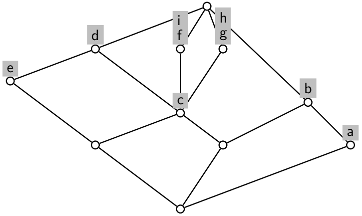
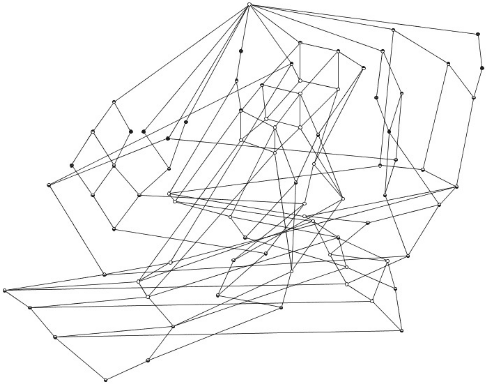
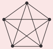


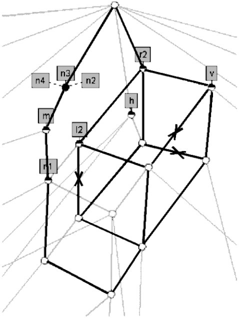


graph of the concept lattice of Pāṇini's *pratyāhāras*

Non-S-encodability of Pāṇini's *pratyāhāras*

Criterion of Kuratowski

A graph which has the graph  as a minor is not planar.



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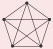
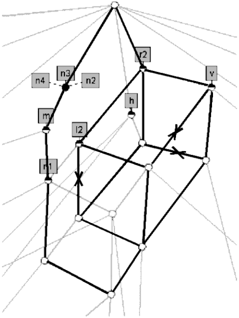
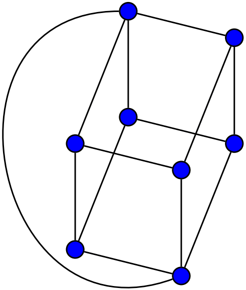
A graph which has the graph  as a minor is not planar.





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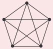
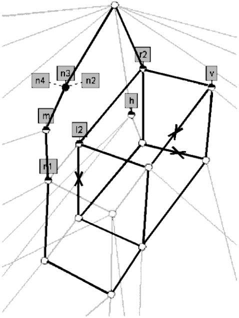
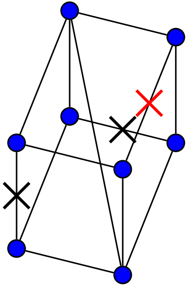
A graph which has the graph  as a minor is not planar.





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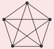
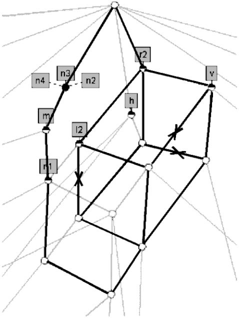
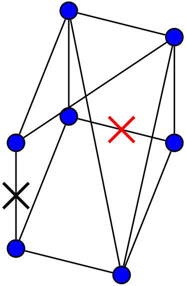
A graph which has the graph  as a minor is not planar.





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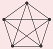
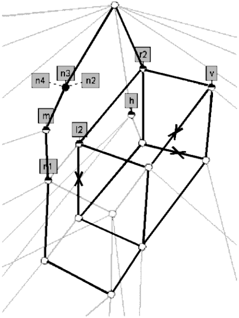
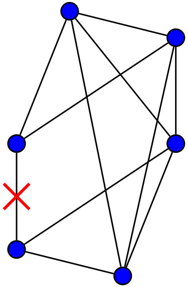
A graph which has the graph  as a minor is not planar.





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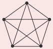
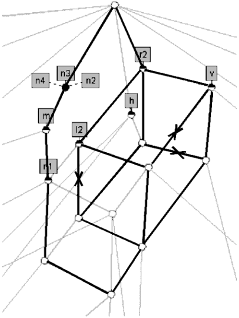
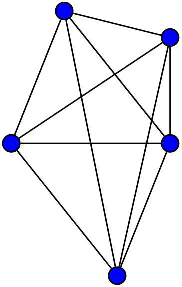
A graph which has the graph  as a minor is not planar.

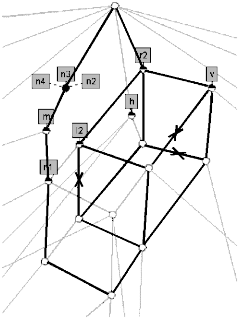


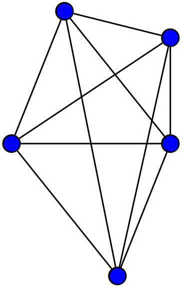


Non-S-encodability of Pāṇini's *pratyāhāras*

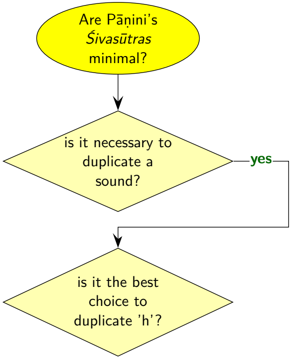
Criterion of Kuratowski

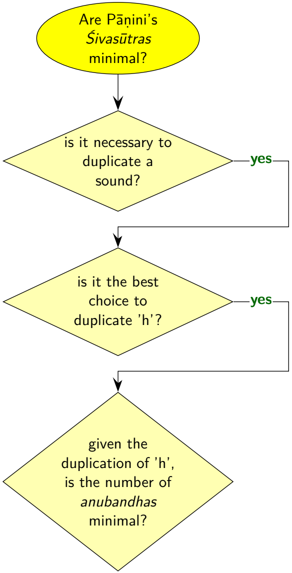
A graph which has the graph  as a minor is not planar.





The set of classes given by Pāṇini's *pratyāhāras* is not S-encodable without duplications!


Are Pāṇini's *Śivasūtras* minimal?
is it necessary to duplicate a sound?
yes
is it the best choice to duplicate 'h'?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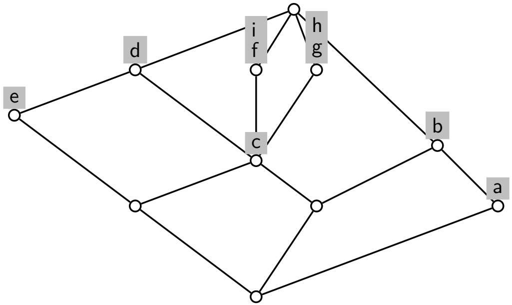
given the duplication of 'h', is the number of *anubandhas* minimal?

S-alphabets with a minimal number of markers



procedure

Start with the empty sequence and choose a walk through the S-graph:

- While moving upwards do nothing.
- While moving downwards along an edge add a new marker to the sequence unless its last element is already a marker.
- If a labeled node is reached, add the labels in arbitrary order to the sequence, unless it has been added before.

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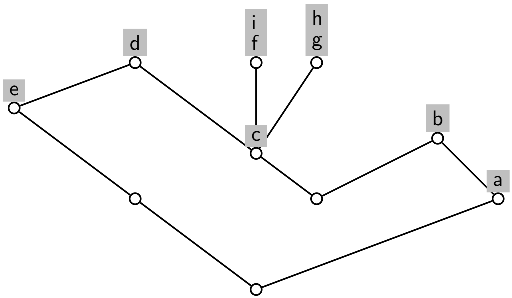

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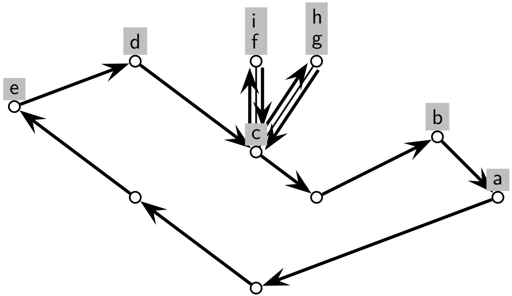

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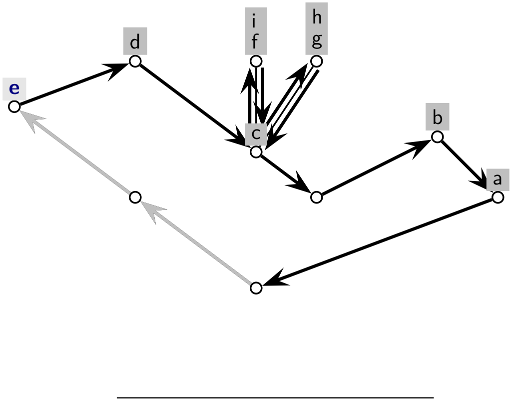

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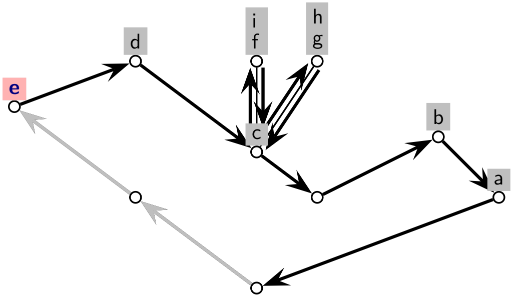

e

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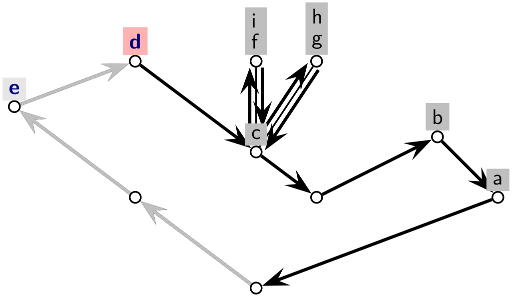

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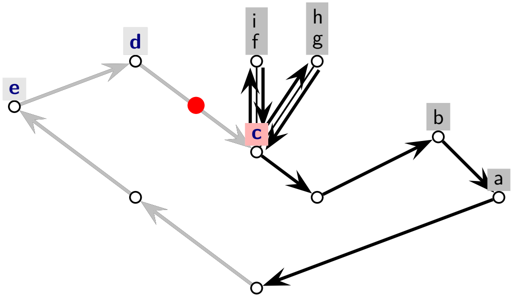

$ed\,M_1\,c$

procedure

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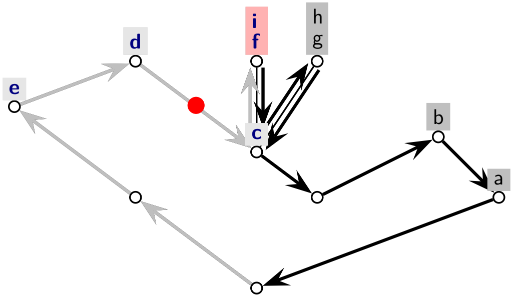

$ed\,M_1\,cfi$

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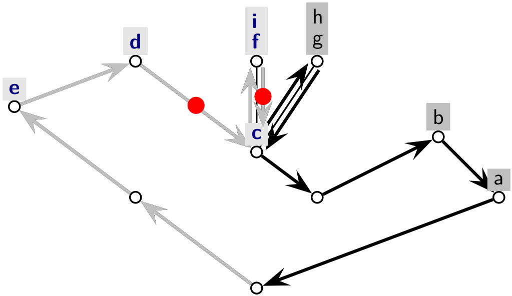

$ed\,M_1\,cfi\,M_2$

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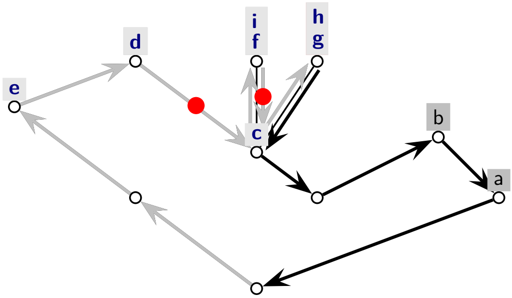

$ed\,M_1\,c\,fi\,M_2\,gh$

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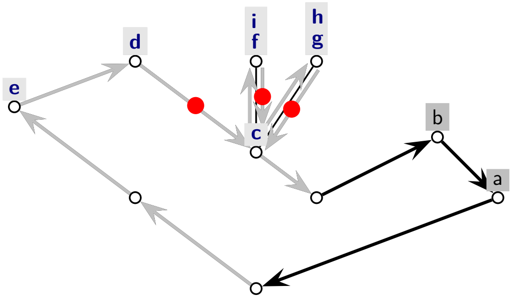

$ed\,M_1\,cfi\,M_2\,gh\,M_3$

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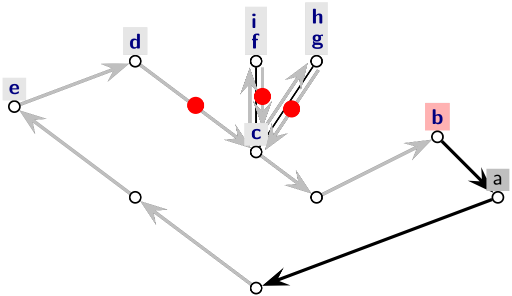

$edM_1cfiM_2ghM_3b$

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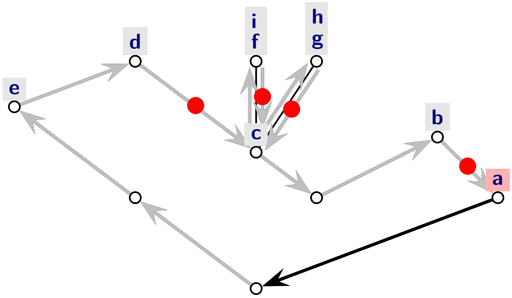

$ed\,M_1\,c\,f\,i\,M_2\,g\,h\,M_3\,b\,M_4\,a$

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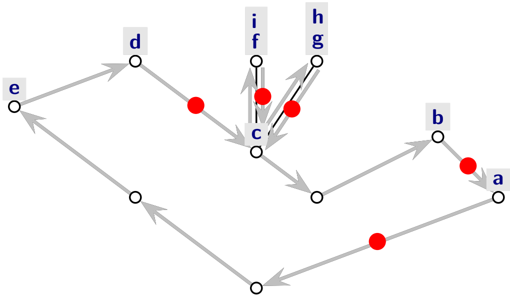

$ed\,M_1\,c\,f\,i\,M_2\,g\,h\,M_3\,b\,M_4\,a\,M_5$

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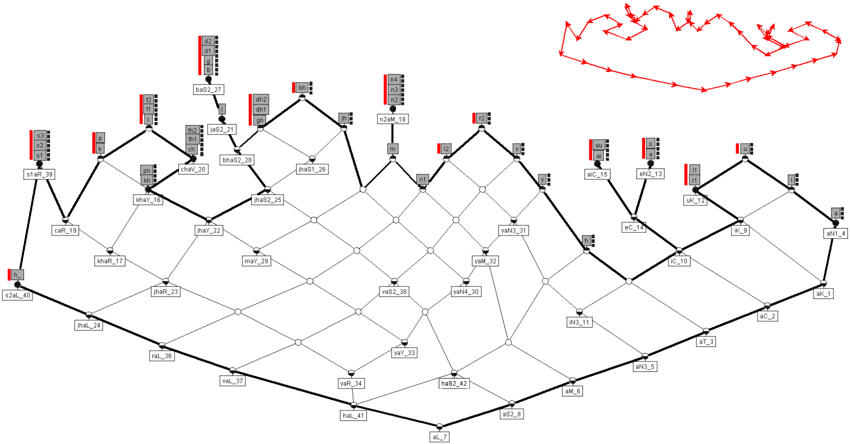
Enlarged concept lattice of Pāṇini's *pratyāhāras*



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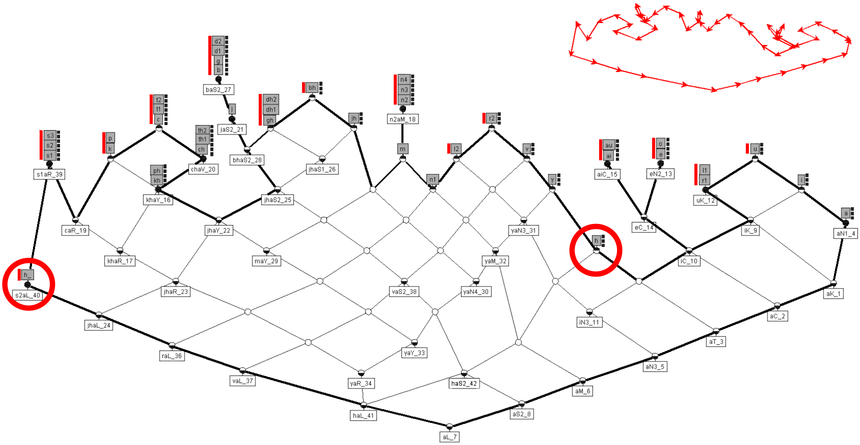

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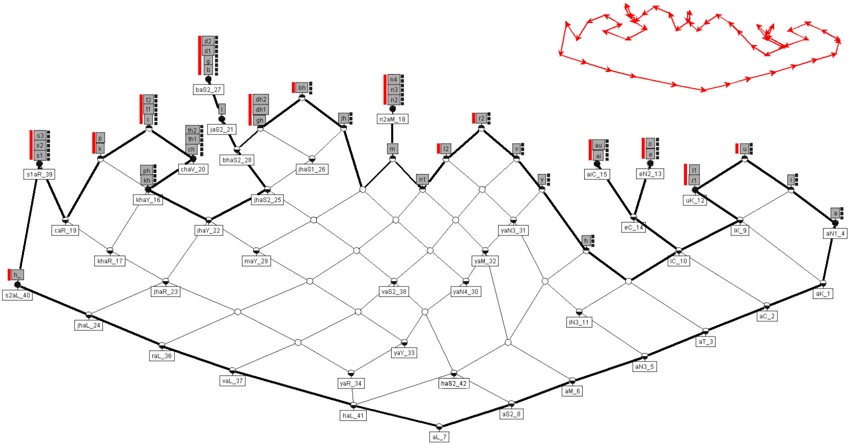
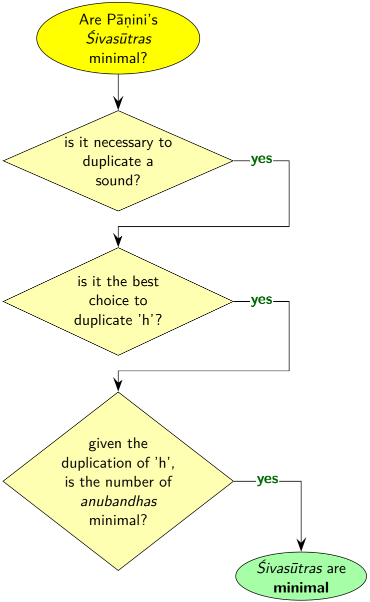


With the *Śivasūtras* Pāṇini has chosen one out of nearly 12 million S-alphabets which fulfill the 5th minimality criterion!


Are Pāṇini's *Śivasūtras* minimal?
is it necessary to duplicate a sound?
yes
is it the best choice to duplicate 'h'?
yes
given the duplication of 'h', is the number of *anubandhas* minimal?
yes
Śivasūtras are **minimal**

The Problem:

Sometimes we are forced to order things (nearly) linearly,
e. g. in . . .

Libraries

Warehouses

Stores

Solution to the problem: revitalize Pāṇini's *Śivasūtra* technique

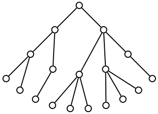
Pāṇini's *Śivasūtra* technique

अइउण्। ऋऌक्। एओङ्। ऐऔच्। हयवरट्।
लण्। ञमङणनम्। झभञ्। घढधष्। जबगडदश्।
खफछठथचटतव्। कपय्। शषसर्। हल्।

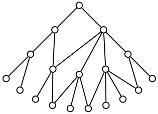
a·i·uṇ | ṛ·ḷk | e·oṅ | ai·auc | hayavaraṭ |
laṇ | ñamaṅaṇanam | jhabhañ | ghaḍhadhaṣ | jabagaḍadaś |
khaphachaṭhathacaṭatav | kapay | śaṣasar | hal |

Transfer

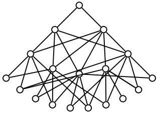
- For physical objects ,duplicating' means ,adding copies'
- Adding copies is annoying but often not impossible
- Ordering objects in an S-order may
  - improve user-friendliness
  - save time
  - save space
  - simplify visual representations of classifications



tree



S-sortable



general hierarchy

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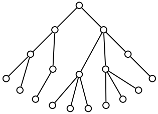

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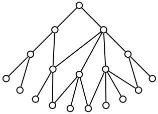

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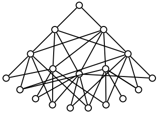
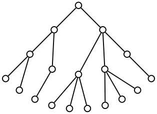


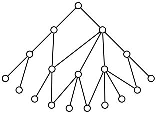
general hierarchy

Transfer

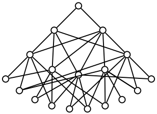
- For physical objects ‚duplicating' means ‚adding copies'
- Adding copies is annoying but often not impossible
- Ordering objects in an S-order may
  - improve user-friendliness
  - save time
  - save space
  - simplify visual representations of classifications



tree



S-sortable



general hierarchy

Transfer

Objects in libraries, ware-houses, and stores are only *nearly* linearly arranged:

⇒ Second (and third) dimension can be used in order to avoid duplications

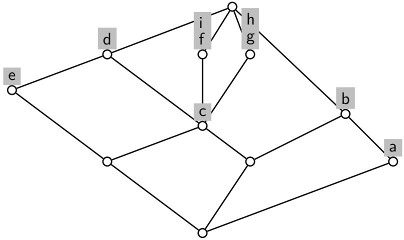
Main theorem of S-sortability

Main theorem on S-encodability

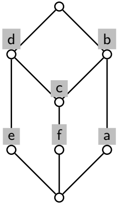
A set of classes $(\mathcal{A}, \Phi)$ is S-encodable without duplications if one of the following equivalent statements is true:

1. The concept lattice of $(\mathcal{A}, \Phi)$ is a Hasse-planar graph and for any $a \in \mathcal{A}$ there is a node labeled a in the S-graph.
2. The concept lattice of the enlarged set of classes $(\mathcal{A}, \tilde{\Phi})$ is Hasse-planar. ($\tilde{\Phi} = \Phi \cup \{\{a\} \mid a \in \mathcal{A}\}$)
3. The Ferrers-graph of the enlarged $(\mathcal{A}, \tilde{\Phi})$-context is bipartite.

Example: S-sortable



Example: not S-sortable



$\{\{d, e\}, \{a, b\}, \{b, c, d\}, \{b, c, d, f\}\}$

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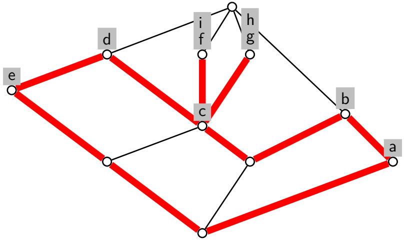

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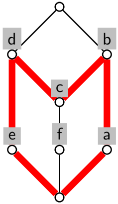

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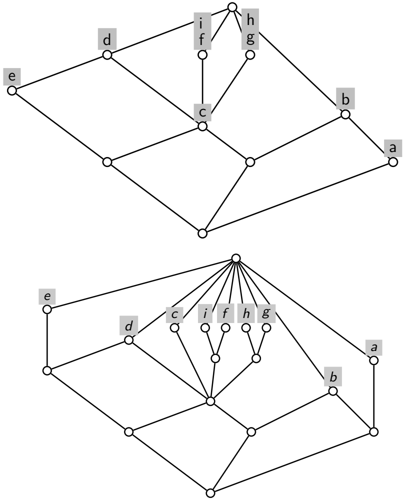

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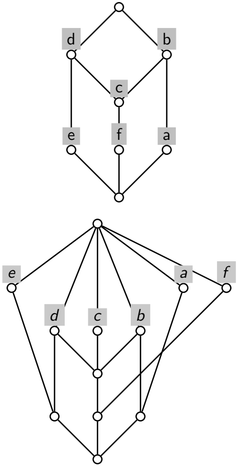

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Advantages:

- The Ferrers-graph is constructed on the formal context.
- Its bipartity can be checked algorithmically.

The story is much more intricate

- We have not shown that Pāṇini's technique for the representation of sound classes is optimal.
- Even we have not shown that he used his technique in an optimal way.

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Literature

- Kiparsky, P. (1991), Economy and the construction of the Śivasūtras. In: M. M. Deshpande & S. Bhate (eds.), *Pāṇinian Studies*, Michigan: Ann Arbor.
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Origin of Pictures

- libraries (left): `http://www.meduniwien.ac.at/medizinischepsychologie/bibliothek.htm`
- libraries (middle): `http://www.math-nat.de/aktuelles/allgemein.htm`
- libraries (right): `http://www.geschichte.mpg.de/deutsch/bibliothek.html`
- warehouses: `http://www.metrogroup.de/servlet/PB/menu/1114920_l1/index.html`
- stores: `http://www.einkaufsparadies-schmidt.de/01bilder01/`