Induction of Classifications from Linguistic Data

speaker: Wiebke Petersen

(Osswald/Petersen 2002)
Example context:
Inflectional paradigms of German nouns

<table>
<thead>
<tr>
<th></th>
<th>gender</th>
<th>sing nom</th>
<th>sing gen</th>
<th>sing dat</th>
<th>sing acc</th>
<th>plur nom</th>
<th>plur gen</th>
<th>plur dat</th>
<th>plur acc</th>
</tr>
</thead>
<tbody>
<tr>
<td>Herr</td>
<td>masc</td>
<td>*</td>
<td>*_n</td>
<td>*_n</td>
<td>*_n</td>
<td>*_n</td>
<td>*_n</td>
<td>*_n</td>
<td>*_n</td>
</tr>
<tr>
<td>Name</td>
<td>masc</td>
<td>*</td>
<td>*_ns</td>
<td>*_n</td>
<td>*_n</td>
<td>*_n</td>
<td>*_n</td>
<td>*_n</td>
<td>*_n</td>
</tr>
<tr>
<td>Staat</td>
<td>masc</td>
<td>*</td>
<td>*_s</td>
<td>*</td>
<td>*_n</td>
<td>*_n</td>
<td>*_n</td>
<td>*_n</td>
<td>*_n</td>
</tr>
<tr>
<td>Hemd</td>
<td>neut</td>
<td>*</td>
<td>*_s</td>
<td>*</td>
<td>*_n</td>
<td>*_n</td>
<td>*_n</td>
<td>*_n</td>
<td>*_n</td>
</tr>
<tr>
<td>Farbe</td>
<td>fem</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*_n</td>
<td>*_n</td>
<td>*_n</td>
<td>*_n</td>
<td>*_n</td>
</tr>
<tr>
<td>Bett</td>
<td>neut</td>
<td>*</td>
<td>*_s</td>
<td>*</td>
<td>*_n</td>
<td>*_n</td>
<td>*_n</td>
<td>*_n</td>
<td>*_n</td>
</tr>
<tr>
<td>Onkel</td>
<td>masc</td>
<td>*</td>
<td>*_s</td>
<td>*</td>
<td>*</td>
<td>*_n</td>
<td>*_n</td>
<td>*_n</td>
<td>*</td>
</tr>
<tr>
<td>Ufer</td>
<td>neut</td>
<td>*</td>
<td>*_s</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*_n</td>
<td>*</td>
<td>*</td>
</tr>
</tbody>
</table>
Example Concept Lattice:

Inflectional paradigms of German nouns
Examples of linguistic classification

And/Or trees (e.g. Koenig, 1999)
Classification as (First Order) Theory

And/OR trees (e.g. Koenig, 1999)

\[
\begin{align*}
A & \subseteq B \text{ stands for } \forall x (Ax \rightarrow Bx) \\
\Lambda & \text{ stands for } \lambda x (x \neq x) \\
V & \text{ stands for } \lambda x (x = x)
\end{align*}
\]

nom \wedge \text{gen } \subseteq \Lambda, \ldots, \text{sing } \wedge \text{plur } \subseteq \Lambda \text{ (ISNOTA)}

nom \lor \text{gen} \lor \text{dat} \lor \text{acc} \subseteq \text{nominal}, \ldots \text{ (ISA)}

\text{nominal } \subseteq \text{nom} \lor \text{gen} \lor \text{dat} \lor \text{acc}, \ldots \text{ (exhaustiveness)}
Examples of linguistic classification

Taxonomic trees (after Eisenberg, 1999)
The canonical universe $C(\Gamma)$

For each observational theory $\Gamma$ over a set of primitive predicates $\Sigma$, there is a canonical model $M(\Gamma) = (C(\Gamma), \models)$, where $X \models p$ iff $p \in X$, for every $X \in C(\Gamma)$ and $p \in \Sigma$. $\models$ is inductively extended to $T[\Gamma]$, the term algebra of observational predicates over $\Sigma$. $C(\Gamma)$ consists of the $\Gamma$-closed consistent subsets of $\Sigma$. 
Construction of the canonical universe

Theory: (Mathematics and linguistics are the only formal sciences)

\[
\text{formal} \land \text{science} \subseteq \text{mathematics} \lor \text{linguistics}
\]

\[
\text{formal} \land \text{science} \supseteq \text{mathematics} \lor \text{linguistics}
\]

Canonical universe:

- mathematical linguistics: \{m, l, f, s\}
- mathematical linguistics: \{m, f, s\}
- formal: \{f\}
- formal: \{s\}
- something: \emptyset

Diagram:

```
  something
     /\   /
    /   /  \\
   /     /    \\
  ∅   formal \{f\} \{s\} science
     /\   /
    /   /  \\
   /     /    \\
  mathematics \{m, f, s\} \{l, f, s\} linguistics
     /\   /
    /   /  \\
   /     /    \\
  {m, f, s} mathematical linguistics
```
Relationship between $\Lambda$-free Horn theories and Concept Lattices

$\Gamma_{\Lambda\text{-free Horn, } M} \leftrightarrow \mathcal{I}_\mathcal{C}$

canonical universe

set of attribute implications

concept lattice
Simple inheritance

no disjunctions, no conjunctions, no $\Lambda$ or $V$

$\Gamma = \{d \subseteq c, c \subseteq a, e \subseteq a, e \subseteq b\}$

$C(\Gamma)$: closed under intersection and union

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>$x_2$</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x_3$</td>
<td></td>
<td>x</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x_4$</td>
<td>x</td>
<td>x</td>
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<td>x</td>
<td></td>
</tr>
<tr>
<td>$x_5$</td>
<td>x</td>
<td>x</td>
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</tr>
<tr>
<td>$x_6$</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>$x_7$</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td></td>
</tr>
</tbody>
</table>
no disjunctions

\[ \Gamma = \{ b \land c \subseteq d, \ d \subseteq c, \ c \subseteq a, \ e \subseteq a, \ e \subseteq b \} \]

\[ C(\Gamma): \text{closed under intersection} \]
Observational Theory

\[ \Gamma = \{ V \subseteq a \lor b, a \land b \subseteq c \lor e, c \land e \subseteq \Lambda, \]
\[ b \land c \subseteq d, d \subseteq c, c \subseteq a, e \subseteq a, e \subseteq b \} \]
Concept lattice $\cong \wedge$-free Horn theory
AOC-poset

sing dat:* ∧ plur nom:*_n ⊆ gender:fem ∨ sing gen:*_s
### Relationship between a theory $\Gamma$ and its canonical universe $C(\Gamma)$

<table>
<thead>
<tr>
<th>Class of $\Gamma$</th>
<th>Closure properties of $C(\Gamma)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>observational</td>
<td>local membership</td>
</tr>
<tr>
<td>Horn</td>
<td>nonempty intersection + directed union</td>
</tr>
<tr>
<td>$\Lambda$-free Horn</td>
<td>intersection + directed union</td>
</tr>
<tr>
<td>simple inheritance</td>
<td>intersection + union</td>
</tr>
<tr>
<td>exclusion</td>
<td>subsets + finitely bounded union</td>
</tr>
<tr>
<td>simple inheritance + exclusion</td>
<td>nonempty intersection + finitely bounded union</td>
</tr>
</tbody>
</table>
Examples of linguistic classification
Systemic networks (e.g. Winograd, 1983)

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