On the minimality of Pāṇini’s Śivasūtras

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### Phonological Rules

**modern notation**

A is replaced by $B$ if preceded by $C$ and succeeded by $D$.

$$A \rightarrow B/C_{-D}$$

**example: final devoicing**

$$\begin{bmatrix}
+ \text{ consonantal} \\
- \text{ nasal} \\
+ \text{ voiced}
\end{bmatrix} \rightarrow \begin{bmatrix}
+ \text{ consonantal} \\
- \text{ nasal} \\
- \text{ voiced}
\end{bmatrix} /\_\# $$
**Phonological Rules**

<table>
<thead>
<tr>
<th>modern notation</th>
</tr>
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<tbody>
<tr>
<td>$A$ is replaced by $B$ if preceded by $C$ and succeeded by $D$.</td>
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$$A \rightarrow B/C_{-}D$$

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<td>$\left[ \begin{array}{c} + \text{ consonantal} \ - \text{ nasal} \ + \text{ voiced} \end{array} \right] \rightarrow \left[ \begin{array}{c} + \text{ consonantal} \ - \text{ nasal} \ - \text{ voiced} \end{array} \right] /__#$</td>
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Modern notation

A is replaced by B if preceded by C and succeeded by D.

\[ A \rightarrow B/C\_D \]

Panini's linear Coding

A + genitive, B + nominative, C + ablative, D + locative.

Example

- Sutra 6.1.77: iko yañaci (इको यणचि)
- Analysis: [ik]_{gen}[yañ]_{nom}[ac]_{loc}
- Modern notation: [iK] \rightarrow [yN]/\_\_ [aC]
Phonological Rules

**modern notation**

A is replaced by $B$ if preceded by $C$ and succeeded by $D$.

$$A \rightarrow B/C\_D$$

**Pāṇini’s linear Coding**

$A +$ genitive, $B +$ nominative, $C +$ ablative, $D +$ locative.

**example**

- *śūtra 6.1.77: iko yaṇacī (इको यणचि)*
- analysis: $[iK]_{gen}[yaṇ]_{nom}[ac]_{loc}$
- modern notation: $[iK] \rightarrow [yN]/\_\_ [aC]$
Pāṇini faced the problem of giving a linear representation of the nonlinear system of sound classes. A similar problem occurs in . . .
Libraries
Warehouses and stores
## Pāṇini’s solution: Śivasūtras

| 1. | a | i | u | Ą |
| 2. | r | ! | K |
| 3. | e | o | Ą |
| 4. | ai | au | C |
| 5. | h | y | v | Ą |
| 6. | l | Ą |
| 7. | ň | m | ň | Ą |
| 8. | jh | bh | Ą |
| 9. | gh | dh | dh | Š |
| 10. | j | b | g | d | Š |
| 11. | kh | ph | c | t | t | V |
| 12. | k | p | Y |
| 13. | š | š | s | R |
| 14. | h | L |

### Wien Petersen

On the minimality of Pāṇini’s Śivasūtras
### Pāṇini’s solution: Śivasūtras

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>i</th>
<th>u</th>
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<tbody>
<tr>
<td>2</td>
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<td>8</td>
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- **AIPA** | **AIPA** | **AIPA** | **AIPA** | **AIPA** |
- a.i.un | r.lk | e.oni | ai.auc | hayavarat | lan | ōn | namanāṇam | jhabhañ | ghadhadhaś | jabagaḍadaś | khaphachathathacatatav | kapay | šaśasar | hal
Pāṇini’s solution: Śivasūtras

<table>
<thead>
<tr>
<th>1.</th>
<th>a</th>
<th>i</th>
<th>u</th>
<th>ānubandha</th>
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Pratyāhāras

1. a i u ň
2. r ! k
3. e o Ň
4. ai au C
5. h y v r Ŧ
### Pratyāhāras

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iK

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Pratyāhāras

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\[ iK = \langle i, u, r, ! \rangle \]
Analysis of iko yañaci: \([iK] \rightarrow [y\text{Ñ}] / \_ / [aC]\)

1. a i u N
2. r ! K
3. e o N
4. ai au C
5. h y v r T
6. l N

\([iK] \rightarrow [y\text{Ñ}] / \_ / [aC]\)

\(\langle i, u, r, l \rangle \rightarrow \langle y, v, r, l \rangle / \_ / \langle a, i, u, r, l, e, o, ai, au \rangle\)
Analysis of iko yaṣaṭaci: \([iK] \rightarrow [yṽ]/_{\_}[aC]\)

1. a i u Ŵ
2. r ! Ŵ K
3. e o Ŵ Ŵ
4. ai au Ŵ C
5. h y v r Ŵ Ŵ
6. l Ŵ Ŵ

\([iK] \rightarrow [yṽ]/_{\_}[aC]\)

\(\langle i, u, r, l\rangle \rightarrow \langle y, v, r, l\rangle/_{\_}\langle a, i, u, r, l, e, o, ai, au\rangle\)
### General problem of S-sortability

Given a set of classes, order the elements of the classes (without duplications) in a linear order (in a list) such that each single class forms a continuous interval with respect to that order.

- The target orders are called **S-orders**
- A set of classes is **S-sortable** if it has an S-order
General problem of Śivasūtra-alphabets (S-alphabets)

Given a set of classes, find an S-order of the elements of the classes. Interrupt this list by markers (anubandhas) such that each single class can be denoted by a sound-marker-pair (pratyāhāra).

Note that every S-order becomes a Śivasūtra-alphabet (S-alphabet) by adding a marker (anubandha) behind each element.

Given the set of classes \(\{\{a, b\}, \{a, b, c\}, \{a, b, c, d\}\}\), the order \(a\ b\ c\ d\) is one of its S-orders and \(a\ M_1\ b\ M_2\ c\ M_3\ d\ M_4\) is one of its S-alphabets.
General problem of Śivasūtra-alphabets (S-alphabets)

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Note that every S-order becomes a Śivasūtra-alphabet (S-alphabet) by adding a marker (anubandha) behind each element.

Given the set of classes \{\{a, b\}, \{a, b, c\}, \{a, b, c, d\}\}, the order \(a \ b \ c \ d\) is one of its S-orders and \(a \ M_1 \ b \ M_2 \ c \ M_3 \ d \ M_4\) is one of its S-alphabets.
Some more Examples

S-sortable example

The set of classes:
\[ \{ \{d, e\}, \{a, b\}, \{b, c, d, f, g, h, i\}, \{f, i\}, \{c, d, e, f, g, h, i\}, \{g, h\} \} \]

is S-sortable;

one of its S-orders is

\[ a \ b \ c \ g \ h \ f \ i \ d \ e \]

non-S-sortable example

The set of classes:
\[ \{ \{a, b\}, \{b, c\}, \{a, c\} \} \]

is not S-sortable.

non-S-sortable example

The set of classes:
\[ \{ \{d, e\}, \{a, b\}, \{b, c, d\}, \{b, c, d, f\} \} \]

is not S-sortable.
Some more Examples

**S-sortable example**
The set of classes:
\{
\{d, e\}, \{a, b\}, \{b, c, d, f, g, h, i\}, \{f, i\}, \{c, d, e, f, g, h, i\}, \{g, h\}\} is S-sortable;
one of its S-orders is
\textcolor{red}{a b c g h f i d e}

**non-S-sortable example**
The set of classes:
\\{
\{a, b\}, \{b, c\}, \{a, c\}\} is not S-sortable.

**non-S-sortable example**
The set of classes:
\\{
\{d, e\}, \{a, b\}, \{b, c, d\}, \{b, c, d, f\}\} is not S-sortable.
### Some more Examples

**S-sortable example**

The set of classes:
\[
\{(d, e), \{a, b\}, \{b, c, d, f, g, h, i\}, \{f, i\}, \{c, d, e, f, g, h, i\}, \{g, h\}\}
\]

is S-sortable;

one of its S-orders is

\[
a b c g h f i d e
\]

**non-S-sortable example**

The set of classes:
\[
\{(a, b), \{b, c\}, \{a, c\}\}
\]

is not S-sortable.

**non-S-sortable example**

The set of classes:
\[
\{(d, e), \{a, b\}, \{b, c, d\}, \{b, c, d, f\}\}
\]

is not S-sortable.
**Some more Examples**

**S-sortable example**

The set of classes:
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\{\{d, e\}, \{a, b\}, \{b, c, d, f, g, h, i\}, \{f, i\}, \{c, d, e, f, g, h, i\}, \{g, h\}\}
\]

is S-sortable;

one of its S-orders is

\[
a \ b \ c \ g \ h \ f \ i \ d \ e
\]

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The set of classes:
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### Some more Examples

#### S-sortable example

The set of classes:
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\]

is S-sortable;

one of its S-orders is
\[a b c g h f i d e\]

#### non-S-sortable example

The set of classes:
\[
\{\{a, b\}, \{b, c\}, \{a, c\}\}
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is not S-sortable.

#### non-S-sortable example

The set of classes:
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Some more Examples

S-sortable example

The set of classes:
\{\{d, e\}, \{a, b\}, \{b, c, d, f, g, h, i\}, \{f, i\}, \{c, d, e, f, g, h, i\}, \{g, h\}\} is S-sortable;
one of its S-orders is
\(a \ b \ c \ g \ h \ f \ i \ d \ e\)

non-S-sortable example

The set of classes:
\{\{a, b\}, \{b, c\}, \{a, c\}\} is not S-sortable.

non-S-sortable example

The set of classes:
\{\{d, e\}, \{a, b\}, \{b, c, d\}, \{b, c, d, f\}\} is not S-sortable.
**Introduction**

Some more Examples

**S-sortable example**

The set of classes:

\[
\{\{d, e\}, \{a, b\}, \{b, c, d, f, g, h, i\}, \{f, i\}, \{c, d, e, f, g, h, i\}, \{g, h\}\}
\]

is S-sortable;

one of its S-orders is

\[
a \ b \ c \ g \ h \ f \ i \ d \ e
\]

**non-S-sortable example**

The set of classes:

\[
\{\{a, b\}, \{b, c\}, \{a, c\}\}
\]

is not S-sortable.

**non-S-sortable example**

The set of classes:

\[
\{\{d, e\}, \{a, b\}, \{b, c, d\}, \{b, c, d, f\}\}
\]

is not S-sortable.

\[
a \ b \ c \ d \ e \quad \text{or} \quad e \ d \ c \ b \ a
\]
Some more Examples

S-sortable example
The set of classes:
\[ \{\{d, e\}, \{a, b\}, \{b, c, d, f, g, h, i\}, \{f, i\}, \{c, d, e, f, g, h, i\}, \{g, h\}\} \] is S-sortable;
one of its S-orders is
\[ abcdgfi\]

non-S-sortable example
The set of classes:
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non-S-sortable example
The set of classes:
\[ \{\{d, e\}, \{a, b\}, \{b, c, d\}, \{b, c, d, f\}\} \] is not S-sortable.
\[ abcd\textcolor{red}{e} \text{ or } e\textcolor{red}{d}cba\]
{{d, e}, {a, b}, {b, c, d, f, g, h, i}, {f, i}, {c, d, e, f, g, h, i}, {g, h}}

concept lattice
Visualize relations

\[
\{ \{ d, e \}, \{ a, b \}, \{ b, c, d, f, g, h, i \}, \{ f, i \}, \{ c, d, e, f, g, h, i \}, \{ g, h \} \}
\]

concept lattice
Visualize relations

\{\{d, e\}, \{a, b\}, \{b, c, d, f, g, h, i\}, \{f, i\}, \{c, d, e, f, g, h, i\}, \{g, h\}\}

\{\{a, b\}, \{b, c\}, \{a, c\}\}

\{\{d, e\}, \{a, b\}, \{b, c, d\}, \{b, c, d, f\}\}
Getting back to Pāṇini’s problem

| ai.un | rlk | e.oni | ai.auc | hayavaraṭ |
| laṇ | ūnmanaṇanaṃ | jhabhaṇ | ghaḍhadhaṣ | jabajagadadaś |
| khaphachathathacataṭatau | kapay | šaṣasar | hal |

Q: Are the Śivasūtras minimal (with respect to length)?
What does minimal mean?

The Śivasūtras are **minimal** if it is **impossible** rearrange the Sanskrit sounds in a new list with *anubandhas* such that

1. each *pratyāhāra* forms an interval ending before an *anubandha*,
2. no sound occurs twice
   
   or one sound occurs twice but less *anubandhas* are needed.

⇒ duplicating a sound is worse than adding *anubandhas*
Common semi-formal argument

Śivasūtras:

\[ aK = \{a, i, u, r, l\}, \quad iK = \{i, u, r, l\} \quad \text{and} \quad uK = \{u, r, l\} \Rightarrow a < i < u < r, l \]  
(taken from Kiparsky 1991) but:

\[ jhL = \{h, s, ś, s, p, k, t, t, c, th, th, ch, ph, kh, d, d, g, b, j, dh, ðh, gh, bh, jh\} \]
\[ jhR = \{s, ś, s, p, k, t, t, c, th, th, ch, ph, kh, d, d, g, b, j, dh, ðh, gh, bh, jh\} \]
\[ jhY = \{p, k, t, t, c, th, th, ch, ph, kh, d, d, g, b, j, dh, ðh, gh, bh, jh\} \]
\[ jhŚ = \{d, ð, g, b, j, dh, ðh, gh, bh, jh\} \quad \text{and} \]
\[ jhŚ = \{dh, ðh, gh, bh, jh\} \]
\[ \Rightarrow h < s, ś, s < p, k, t, t, c, th, th, ch, ph, kh, d < ð, g, b, j < dh, ðh, gh, bh, jh \]
### Common semi-formal argument

#### Śivasūtras:

\[
a K = \{a, i, u, r, !\}, \quad i K = \{i, u, r, !\} \text{ and } \quad u K = \{u, r, !\} \Rightarrow a < i < u < r,!
\]

(taken from Kiparsky 1991)

**but:**

\[
jh L = \\
\{h, s, ś, ś, p, k, t, t, c, t h, t h, ch, ph, kh, d, ā, g, b, j, dh, ñh, gh, bh, jh\}
\]

\[
jh R = \\
\{s, ś, ś, p, k, t, t, c, t h, t h, ch, ph, kh, d, ā, g, b, j, dh, ñh, gh, bh, jh\}
\]

\[
jh Y = \{p, k, t, t, c, t h, t h, ch, ph, kh, d, ā, g, b, j, dh, ñh, gh, bh, jh\}
\]

\[
jh ĺ = \{d, ā, g, b, j, dh, ñh, gh, bh, jh\}\]

\[
jh Š = \{dh, ñh, gh, bh, jh\}\]

\[\Rightarrow h < s, ś, ś < p, k, t, t, c, t h, t h, ch, ph, kh, d < ā, g, b, j < dh, ñh, gh, bh, jh\]
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<th>Śivasūtras and pratyāhāras</th>
<th>S-sortability</th>
<th>Minimality of the Śivasūtras</th>
<th>Outlook</th>
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Are Pāṇini’s Śivasūtras minimal?
Are Pāṇini’s Śivasūtras minimal?

Is it necessary to duplicate a sound?
Introduction

Śivasūtras and pratyāhāras

S-sortability

Minimality of the Śivasūtras

Outlook

Are Pāṇini’s Śivasūtras minimal?

- no
  - is it necessary to duplicate a sound?
  - yes
    - is it the best choice to duplicate 'h'?

Śivasūtras are not minimal
Śivasūtras and pratyāhāras

S-sortability

Minimality of the Śivasūtras

Outlook

Introduction

Śivasūtras are not minimal.

Are Pāṇini’s Śivasūtras minimal?

is it necessary to duplicate a sound?

no

is it the best choice to duplicate ‘h’?

no

given the duplication of ‘h’, is the number of anubandhas minimal?

yes

yes

no

yes

no
Are Pāṇini’s Śivasūtras minimal?

- is it necessary to duplicate a sound?
  - no
  - yes

- is it the best choice to duplicate 'h'?
  - no
  - yes

- given the duplication of 'h', is the number of anubandhas minimal?
  - no
  - yes

Śivasūtras are not minimal

Śivasūtras are minimal
Is it necessary to duplicate a sound?

Main theorem on S-sortability (part 1a)

If a set of classes is S-sortable, then its concept lattice is Hasse-planar.
Is it necessary to duplicate a sound?

Main theorem on S-sortability (part 1a)

If a set of classes is S-sortable, then its concept lattice is Hasse-planar.

graph of the concept lattice of Pāṇini’s pratyāhāras
No S-alphabet without duplications for Pāṇini’s *pratyāhāras*

**Criterion of Kuratowski**

A graph which has the graph \( \begin{array}{c} \text{graph} \end{array} \) as a minor is not planar.
No S-alphabet without duplications for Pāṇini’s pratyāhāras

Criterion of Kuratowski

A graph which has the graph \( \begin{tikzpicture} \end{tikzpicture} \) as a minor is not planar.
No S-alphabet without duplications for Pāṇini’s *pratyāhāras*

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No S-alphabet without duplications for Pāṇini’s pratyāhāras

Criterion of Kuratowski

A graph which has the graph as a minor is not planar.
No S-alphabet without duplications for Pāṇini’s *pratyāhāras*

**Criterion of Kuratowski**

A graph which has the graph as a minor is not planar.
No S-alphabet without duplications for Pāṇini’s *pratyāhāras*

**Criterion of Kuratowski**

A graph which has the graph \( \begin{array}{c} \node1 \node2 \node3 \node4 \node5 \node6 \node7 \end{array} \) as a minor is not planar.
No S-alphabet without duplications for Pāṇini’s *pratyāhāras*

**Criterion of Kuratowski**

A graph which has the graph \( K_5 \) as a minor is not planar.

There is no S-alphabet for the set of classes given by Pāṇini’s *pratyāhāras* without duplicated elements!
Are Pāṇini’s Śivasūtras minimal?

- Is it necessary to duplicate a sound? [yes]

- Is it the best choice to duplicate 'h'?
Are Pāṇini’s Śivasūtras minimal?

- Is it necessary to duplicate a sound? (Yes)

- Is it the best choice to duplicate 'h'? (Yes)

- Given the duplication of 'h', is the number of anubandhas minimal?
S-alphabets with a minimal number of markers

**procedure**

Start with the empty sequence and choose a walk through the S-graph:

- While moving upwards do nothing.
- While moving downwards along an edge add a new marker to the sequence unless its last element is already a marker.
- If a labeled node is reached, add the labels in arbitrary order to the sequence, unless it has been added before.
S-alphabets with a minimal number of markers

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**procedure**

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procedure

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**procedure**

Start with the empty sequence and choose a walk through the S-graph:

- **While moving upwards**
  - do nothing.

- **While moving downwards along an edge** add a new marker to the sequence unless its last element is already a marker.

- **If a sound is reached,**
  - add the sound to the sequence, unless it has been added before.
S-alphabets with a minimal number of markers

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\[ edM_1cfi \]
S-alphabets with a minimal number of markers

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\[ed M_1 cfi M_2\]
procedure

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\[ edM_1cfiM_2gh \]
**S-alphabets with a minimal number of markers**

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\[ ed \ M_1 \ cfi \ M_2 \ gh \ M_3 \]
S-alphabets with a minimal number of markers

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\[ed M_1 cfi M_2 gh M_3 b\]
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\[ ed M_1 c f i M_2 g h M_3 b M_4 a \]
S-alphabets with a minimal number of markers

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\[
ed M_1 c f i M_2 gh M_3 b M_4 a M_5\]
Concept lattice of Pāṇini’s *pratyāhāras* with duplicated *h*
Concept lattice of Pāṇini’s pratyāhāras with duplicated $h$
Introduction

Śivasūtras and pratyāhāras

S-sortability

Minimality of the Śivasūtras

Outlook

Concept lattice of Pāṇini’s pratyāhāras with duplicated $h$

With the Śivasūtras Pāṇini has chosen one out of nearly 12 million minimal S-alphabets!
Are Pāṇini’s Śivasūtras minimal?

- Is it necessary to duplicate a sound? 
  - Yes

- Is it the best choice to duplicate 'h'? 
  - Yes

- Given the duplication of 'h', is the number of anubandhas minimal? 
  - Yes

Śivasūtras are minimal

---

On the minimality of Pāṇini’s Śivasūtras

Wiebke Petersen
Transfer

- For physical objects, ‘duplication’ means ‘adding copies’
- Adding copies is annoying but often not impossible
- Ordering objects in an S-order may
  - improve user-friendliness
  - save time
  - save space
  - simplify visual representations of classifications

![Tree](image1)
![S-sortable](image2)
![General hierarchy](image3)

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- tree
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  - simplify visual representations of classifications

Tree

S-sortable

General hierarchy
Transfer

Objects in libraries, ware-houses, and stores are only *nearly* linearly arranged:

⇒ Second (and third) dimension can be used in order to avoid duplications
What explains the actual structure of the Śivasūtras?

- principle of homorganic continuity (Staal, 1962)
- principle of historic continuity (Cardona, 1969)
- principle of economy and logic of the special case and the general case (Kiparsky 1991) or Pāṇini’s razor (Kiparsky 2007)

The presented approach cannot give an answer to this question.

The story is much more intricate

- We have neither shown that Pāṇini’s technique for the representation of sound classes is optimal
- nor that he has used his technique in an optimal way.
Open problems

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Origin of Pictures

- libraries (left): http://www.meduniwien.ac.at/medizinischepsychologie/bibliothek.htm
- libraries (middle): http://www.math-nat.de/aktuelles/allgemein.htm
- libraries (right):
  http://www.geschichte.mpg.de/deutsch/bibliothek.html
- warehouses:
  http://www.metrogroup.de/servlet/PB/menu/1114920_l1/index.html
- stores: http://www.einkaufsparadies-schmidt.de/01bilder01/