On the minimality of Pāṇini's *Śivasūtras*

Wiebke Petersen

Institute of Language and Information
University of Düsseldorf, Germany
petersew@uni-duesseldorf.de

University of English and Foreign Languages, 19. January 2009

अइउण्। ऋऌक्। एओङ्। ऐऔच्। हयवरट्।
लण्। ञमङणनम्। झभञ्। घढधष्। जबगडदश्।
खफछठथचटतव्। कपय्। शषसर्। हल्।

Phonological Rules

modern notation

A is replaced by B if preceded by C and succeeded by D.

$$A \rightarrow B/_{C\_\_D}$$

example: final devoicing

$$\begin{bmatrix} + & \text{consonantal} \\ - & \text{nasal} \\ + & \text{voiced} \end{bmatrix} \rightarrow \begin{bmatrix} + & \text{consonantal} \\ - & \text{nasal} \\ - & \text{voiced} \end{bmatrix} /\_\_\sharp$$

Phonological Rules

modern notation

A is replaced by B if preceded by C and succeeded by D.

$$A \rightarrow B/_{C\_\_D}$$

example: final devoicing

$$\begin{bmatrix} + & \text{consonantal} \\ - & \text{nasal} \\ + & \text{voiced} \end{bmatrix} \rightarrow \begin{bmatrix} + & \text{consonantal} \\ - & \text{nasal} \\ - & \text{voiced} \end{bmatrix} /\_\_\sharp$$

Phonological Rules

modern notation

A is replaced by B if preceded by C and succeeded by D.

$$A \rightarrow B/_{C\_\_D}$$

Pāṇini's linear Coding

$$A + \text{genitive},\ B + \text{nominative},\ C + \text{ablative},\ D + \text{locative}.$$

example

- *sūtra* 6.1.77: *iko yaṇaci* (इको यणचि)
- analysis: [ik]$_{\text{gen}}$[yaṇ]$_{\text{nom}}$[ac]$_{\text{loc}}$
- modern notation: [iK] → [yṆ]/__[aC]

Phonological Rules

modern notation

A is replaced by B if preceded by C and succeeded by D.

$$A \rightarrow B/_{C\_\_D}$$

Pāṇini's linear Coding

$$A + \text{genitive}, B + \text{nominative}, C + \text{ablative}, D + \text{locative}.$$

example

- *sūtra* 6.1.77: *iko yaṇaci* (इको यणचि)
- analysis: [ik]$_{\text{gen}}$[yaṇ]$_{\text{nom}}$[ac]$_{\text{loc}}$
- modern notation: [iK] → [yṆ]/__ [aC]

Pāṇini faced the problem of giving a linear representation of the nonlinear system of sound classes.

A similar problem occurs in . . .

Libraries

Warehouses and stores

Pāṇini's solution: *Śivasūtras*

1.	a	i	u			Ṇ
2.				ṛ	ḷ	K
3.		e	o			Ṅ
4.		ai	au			C
5.	h	y	v	r		Ṭ
6.					l	Ṇ
7.	ñ	m	ṅ	ṇ	n	M
8.	jh	bh				Ñ
9.			gh	ḍh	dh	Ṣ
10.	j	b	g	ḍ	d	Ś
11.	kh	ph	ch	ṭh	th	
			c	ṭ	t	V
12.	k	p				Y
13.		ś	ṣ	s		R
14.	h					L

अइउण्। ऋऌक्।
a·i·uṇ| *ṛ·ḷk*|

एओङ्। ऐऔच्।
e·oṅ| *ai·auc*|

हयवरट्। लण्।
hayavaraṭ| *laṇ*|

ञमङणनम्। झभञ्।
ñamaṅaṇanam| *jhabhañ*|

घढधष्। जबगडदश्।
ghaḍhadhaṣ| *jabagaḍadaś*|

खफछठथचटतव्।
khaphachaṭhathacaṭatav|

कपय्। शषसर्। हल्।
kapay| *śaṣasar*| *hal*|

Pāṇini's solution: *Śivasūtras*

1.	a	i	u			Ṇ
2.				ṛ	ḷ	K
3.		e	o			Ṅ
4.		ai	au			C
5.	h	y	v	r		Ṭ
6.					l	Ṇ
7.	ñ	m	ṅ	ṇ	n	M
8.	jh	bh				Ñ
9.			gh	ḍh	dh	Ṣ
10.	j	b	g	ḍ	d	Ś
11.	kh	ph	ch	ṭh	th	
			c	ṭ	t	V
12.	k	p				Y
13.		ś	ṣ	s		R
14.	h					L

अइउण्। ऋऌक्।
a·i·uṇ| ṛ·ḷk|

एओङ्। ऐऔच्।
e·oṅ| ai·auc|

हयवरट्। लण्।
hayavaraṭ| laṇ|

ञमङणनम्। झभञ्।
ñamaṅaṇanam| jhabhañ|

घढधष्। जबगडदश्।
ghaḍhadhaṣ| jabagaḍadaś|

खफछठथचटतव्।
khaphachaṭhathacaṭatav|

कपय्। शषसर्। हल्।
kapay| śaṣasar| hal|

Pāṇini's solution: *Śivasūtras*

1.	a	i	u			Ṇ
2.				ṛ	ḷ	K
3.		e	o			Ṅ
4.		ai	au			C
5.	h	y	v	r		Ṭ
6.					l	Ṇ
7.	ñ	m	ṅ	ṇ	n	M
8.	jh	bh				Ñ
9.			gh	ḍh	dh	Ṣ
10.	j	b	g	ḍ	d	Ś
11.	kh	ph	ch	ṭh	th	
			c	ṭ	t	V
12.	k	p				Y
13.		ś	ṣ	s		R
14.	h					L

anubandha

अइउण्। ऋऌक्।
a·i·uṇ| ṛ·ḷk|
एओङ्। ऐऔच्।
e·oṅ| ai·auc|
हयवरट्। लण्।
hayavaraṭ| laṇ|
ञमङणनम्। झभञ्।
ñamaṅaṇanam| jhabhañ|
घढधष्। जबगडदश्।
ghaḍhadhaṣ| jabagaḍadaś|
खफछठथचटतव्।
khaphachaṭhathacaṭatav|
कपय्। शषसर्। हल्।
kapay| śaṣasar| hal|

Pratyāhāras

1.	a	i	u			Ṇ
2.				ṛ	ḷ	K
3.		e	o			Ṅ
4.		ai	au			C
5.	h	y	v	r		Ṭ

Pratyāhāras

1.	a	i	u			Ṇ
2.				ṛ	ḷ	K
3.		e	o			Ṅ
4.		ai	au			C
5.	h	y	v	r		Ṭ

iK

Pratyāhāras

1.	a	i	u			Ṇ
2.				ṛ	ḷ	K
3.		e	o			Ṅ
4.		ai	au			C
5.	h	y	v	r		Ṭ

iK= ⟨i, u, ṛ, ḷ⟩

Analysis of iko yaṇaci: [iK] → [yṆ]/_ [aC]

1.	a	i	u			Ṇ
2.				ṛ	ḷ	K
3.		e	o			Ṅ
4.		ai	au			C
5.	h	y	v	r		Ṭ
6.					l	Ṇ

- [iK] → [yṆ]/_ [aC]
- ⟨i, u, ṛ, ḷ⟩ → ⟨y, v, r, l⟩/_ ⟨a, i, u, ṛ, ḷ, e, o, ai, au⟩

Analysis of iko yaṇaci: [iK] → [yṆ]/__ [aC]

1.	a	i	u			Ṇ
2.				ṛ	ḷ	K
3.		e	o			Ṅ
4.		ai	au			C
5.	h	y	v	r		Ṭ
6.					l	Ṇ

- [iK] → [yṆ]/__ [aC]
- ⟨i, u, ṛ, ḷ⟩ → ⟨y, v, r, l⟩/__ ⟨a, i, u, ṛ, ḷ, e, o, ai, au⟩

General problem of S-sortability

Given a set of classes, order the elements of the classes (without duplications) in a linear order (in a list) such that each single class forms a continuous interval with respect to that order.

- The target orders are called S-orders
- A set of classes is S-sortable if it has an S-order

General problem of *Śivasūtra*-alphabets (S-alphabets)

Given a set of classes, find an S-order of the elements of the classes. Interrupt this list by markers (*anubandhas*) such that each single class can be denoted by a sound-marker-pair (*pratyāhāra*).

Note that every S-order becomes a *Śivasūtra*-alphabet (S-alphabet) by adding a marker (*anubandha*) behind each element.

Given the set of classes $\{\{a, b\}, \{a, b, c\}, \{a, b, c, d\}\}$, the order $a\ b\ c\ d$ is one of its S-orders and $a\ M_1\ b\ M_2\ c\ M_3\ d\ M_4$ is one of its S-alphabets.

General problem of *Śivasūtra*-alphabets (S-alphabets)

Given a set of classes, find an S-order of the elements of the classes. Interrupt this list by markers (*anubandhas*) such that each single class can be denoted by a sound-marker-pair (*pratyāhāra*).

Note that every S-order becomes a *Śivasūtra*-alphabet (S-alphabet) by adding a marker (*anubandha*) behind each element.

Given the set of classes $\{\{a, b\}, \{a, b, c\}, \{a, b, c, d\}\}$, the order $a\ b\ c\ d$ is one of its S-orders and $a\ M_1\ b\ M_2\ c\ M_3\ d\ M_4$ is one of its S-alphabets.

Some more Examples

S-sortable example

The set of classes:
$\{\{d,e\},\{a,b\},\{b,c,d,f,g,h,i\},\{f,i\},\{c,d,e,f,g,h,i\},\{g,h\}\}$ is S-sortable;
one of its S-orders is
$a\,b\,c\,g\,h\,f\,i\,d\,e$

non-S-sortable example

The set of classes:
$\{\{a,b\},\{b,c\},\{a,c\}\}$ is not S-sortable.

non-S-sortable example

The set of classes:
$\{\{d,e\},\{a,b\},\{b,c,d\},\{b,c,d,f\}\}$ is not S-sortable.

Some more Examples

S-sortable example

The set of classes:
$\{\{d, e\}, \{a, b\}, \{b, c, d, f, g, h, i\}, \{f, i\}, \{c, d, e, f, g, h, i\}, \{g, h\}\}$ is S-sortable;
one of its S-orders is
$a\,b\,c\,g\,h\,f\,i\,d\,e$

non-S-sortable example

The set of classes:
$\{\{a, b\}, \{b, c\}, \{a, c\}\}$ is not S-sortable.

non-S-sortable example

The set of classes:
$\{\{d, e\}, \{a, b\}, \{b, c, d\}, \{b, c, d, f\}\}$ is not S-sortable.

Some more Examples

S-sortable example

The set of classes:
$\{\{d,e\},\{a,b\},\{b,c,d,f,g,h,i\},\{f,i\},\{c,d,e,f,g,h,i\},\{g,h\}\}$ is S-sortable;
one of its S-orders is
$a\,b\,c\,g\,h\,f\,i\,d\,e$

non-S-sortable example

The set of classes:
$\{\{a,b\},\{b,c\},\{a,c\}\}$ is not S-sortable.

non-S-sortable example

The set of classes:
$\{\{d,e\},\{a,b\},\{b,c,d\},\{b,c,d,f\}\}$ is not S-sortable.

Some more Examples

S-sortable example

The set of classes:
$\{\{d,e\},\{a,b\},\{b,c,d,f,g,h,i\},\{f,i\},\{c,d,e,f,g,h,i\},\{g,h\}\}$ is S-sortable;
one of its S-orders is
$a\,b\,c\,g\,h\,f\,i\,d\,e$

non-S-sortable example

The set of classes:
$\{\{a,b\},\{b,c\},\{a,c\}\}$ is not S-sortable.

non-S-sortable example

The set of classes:
$\{\{d,e\},\{a,b\},\{b,c,d\},\{b,c,d,f\}\}$ is not S-sortable.

Some more Examples

S-sortable example

The set of classes:
$\{\{d, e\}, \{a, b\}, \{b, c, d, f, g, h, i\}, \{f, i\}, \{c, d, e, f, g, h, i\}, \{g, h\}\}$ is S-sortable;
one of its S-orders is
$a\,b\,c\,g\,h\,f\,i\,d\,e$

non-S-sortable example

The set of classes:
$\{\{a, b\}, \{b, c\}, \{a, c\}\}$ is not S-sortable.

non-S-sortable example

The set of classes:
$\{\{d, e\}, \{a, b\}, \{b, c, d\}, \{b, c, d, f\}\}$ is not S-sortable.

Some more Examples

S-sortable example

The set of classes:
$\{\{d,e\},\{a,b\},\{b,c,d,f,g,h,i\},\{f,i\},\{c,d,e,f,g,h,i\},\{g,h\}\}$ is S-sortable;
one of its S-orders is
$a\,b\,c\,g\,h\,f\,i\,d\,e$

non-S-sortable example

The set of classes:
$\{\{a,b\},\{b,c\},\{a,c\}\}$ is not S-sortable.

non-S-sortable example

The set of classes:
$\{\{d,e\},\{a,b\},\{b,c,d\},\{b,c,d,f\}\}$ is not S-sortable.

Some more Examples

S-sortable example

The set of classes:
$\{\{d,e\},\{a,b\},\{b,c,d,f,g,h,i\},\{f,i\},\{c,d,e,f,g,h,i\},\{g,h\}\}$ is S-sortable;
one of its S-orders is
$a\,b\,c\,g\,h\,f\,i\,d\,e$

non-S-sortable example

The set of classes:
$\{\{a,b\},\{b,c\},\{a,c\}\}$ is not S-sortable.

non-S-sortable example

The set of classes:
$\{\{d,e\},\{a,b\},\{b,c,d\},\{b,c,d,f\}\}$ is not S-sortable.
$a\,b\,c\,d\,e$ or $e\,d\,c\,b\,a$

Some more Examples

S-sortable example

The set of classes:
$\{\{d,e\},\{a,b\},\{b,c,d,f,g,h,i\},\{f,i\},\{c,d,e,f,g,h,i\},\{g,h\}\}$ is S-sortable;
one of its S-orders is
$a\ b\ c\ g\ h\ f\ i\ d\ e$

non-S-sortable example

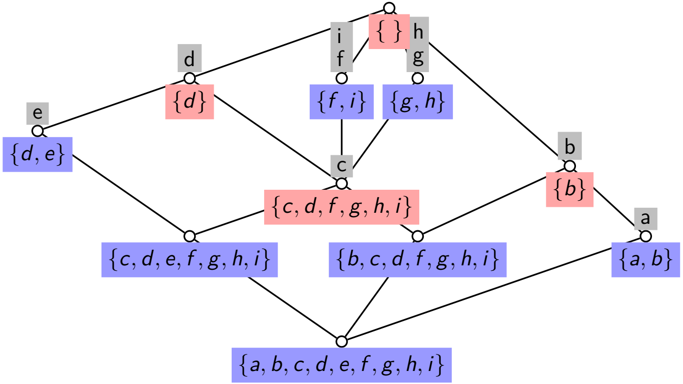
The set of classes:
$\{\{a,b\},\{b,c\},\{a,c\}\}$ is not S-sortable.

non-S-sortable example

The set of classes:
$\{\{d,e\},\{a,b\},\{b,c,d\},\{b,c,d,f\}\}$ is not S-sortable.
$a\ b\ c\ d\ e$ or $e\ d\ c\ b\ a$

Visualize relations

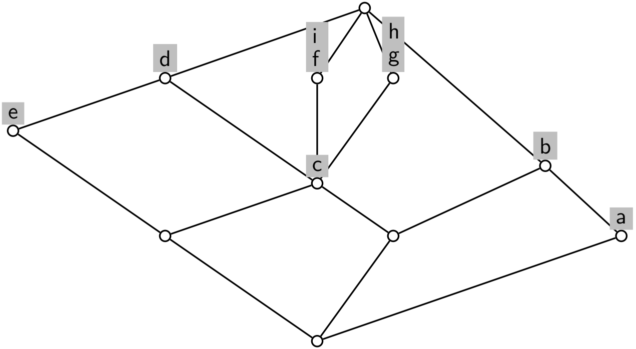
$\{\{d,e\},\{a,b\},\{b,c,d,f,g,h,i\},\{f,i\},\{c,d,e,f,g,h,i\},\{g,h\}\}$



concept lattice

Visualize relations

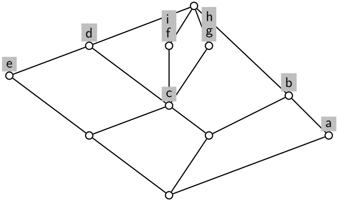
$$\{\{d,e\},\{a,b\},\{b,c,d,f,g,h,i\},\{f,i\},\{c,d,e,f,g,h,i\},\{g,h\}\}$$



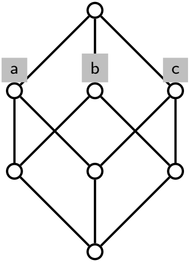
concept lattice

Visualize relations

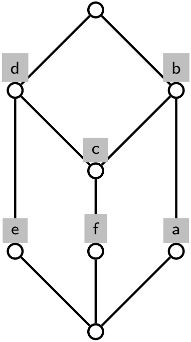
$\{\{d, e\}, \{a, b\}, \{b, c, d, f, g, h, i\}, \{f, i\}, \{c, d, e, f, g, h, i\}, \{g, h\}\}$



$\{\{a, b\}, \{b, c\}, \{a, c\}\}$



$\{\{d, e\}, \{a, b\}, \{b, c, d\}, \{b, c, d, f\}\}$



Getting back to Pāṇini's problem

a·i·uṇ| *ṛ·ḷk*| *e·oṅ*| *ai·auc*| *hayavaraṭ*|
laṇ| *ñamaṅaṇanam*| *jhabhañ*| *ghaḍhadhaṣ*| *jabagaḍadaś*|
khaphachaṭhathacaṭatav| *kapay*| *śaṣasar*| *hal*|

Q: Are the *Śivasūtras* minimal (with respect to length)?

What does minimal mean?

a·i·uṇ| *ṛ·ḷk*| *e·oṅ*| *ai·auc*| *hayavaraṭ*|
laṇ| *ñamaṅaṇanam*| *jhabhañ*| *ghaḍhadhaṣ*| *jabagaḍadaś*|
khaphachaṭhathacaṭatav| *kapay*| *śaṣasar*| *hal*|

The *Śivasūtras* are minimal if it is **im**possible rearrange the Sanskrit sounds in a new list with *anubandhas* such that

1. each *pratyāhāra* forms an interval ending before an *anubandha*,
2. no sound occurs twice

or one sound occurs twice but less *anubandhas* are needed.

⇒ duplicating a sound is worse than adding *anubandhas*

Common semi-formal argument

***Śivasūtras*:**

a i u Ṇ ṛ ḷ K e o Ṅ ai au C h y v r Ṭ l Ṇ ñ m ṅ ṇ n M jh bh Ñ gh ḍh dh Ṣ j b g ḍ d Ś kh ph ch ṭh th c ṭ t V k p Y ś ṣ s R h L

$aK = \{a, i, u, ṛ, ḷ\}$, $iK = \{i, u, ṛ, ḷ\}$ and $uK = \{u, ṛ, ḷ\} \Rightarrow a < i < u < ṛ, ḷ$ (taken from Kiparsky 1991) but:

$jhL = \{h, s, ṣ, ś, p, k, t, ṭ, c, th, ṭh, ch, ph, kh, d, ḍ, g, b, j, dh, ḍh, gh, bh, jh\}$

$jhR = \{s, ṣ, ś, p, k, t, ṭ, c, th, ṭh, ch, ph, kh, d, ḍ, g, b, j, dh, ḍh, gh, bh, jh\}$

$jhY = \{p, k, t, ṭ, c, th, ṭh, ch, ph, kh, d, ḍ, g, b, j, dh, ḍh, gh, bh, jh\}$

$jhŚ = \{d, ḍ, g, b, j, dh, ḍh, gh, bh, jh\}$ and

$jhṢ = \{dh, ḍh, gh, bh, jh\}$

$\Rightarrow h < s, ṣ, ś < p, k, t, ṭ, c, th, ṭh, ch, ph, kh, d < ḍ, g, b, j < dh, ḍh, gh, bh, jh$

Common semi-formal argument

***Śivasūtras*:**

a i u Ṇ ṛ ḷ K e o Ṅ ai au C h y v r Ṭ l Ṇ ñ m ṅ ṇ n M jh bh Ñ gh ḍh dh Ṣ
j b g ḍ d Ś kh ph ch ṭh th c ṭ t V k p Y ś ṣ s R h L

$aK = \{a, i, u, ṛ, ḷ\}$, $iK = \{i, u, ṛ, ḷ\}$ and $uK = \{u, ṛ, ḷ\} \Rightarrow a < i < u < ṛ,ḷ$ (taken from Kiparsky 1991) but:

$jhL =$
$\{h, s, ṣ, ś, p, k, t, ṭ, c, th, ṭh, ch, ph, kh, d, ḍ, g, b, j, dh, ḍh, gh, bh, jh\}$

$jhR =$
$\{s, ṣ, ś, p, k, t, ṭ, c, th, ṭh, ch, ph, kh, d, ḍ, g, b, j, dh, ḍh, gh, bh, jh\}$

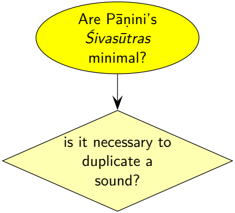
$jhY = \{p, k, t, ṭ, c, th, ṭh, ch, ph, kh, d, ḍ, g, b, j, dh, ḍh, gh, bh, jh\}$

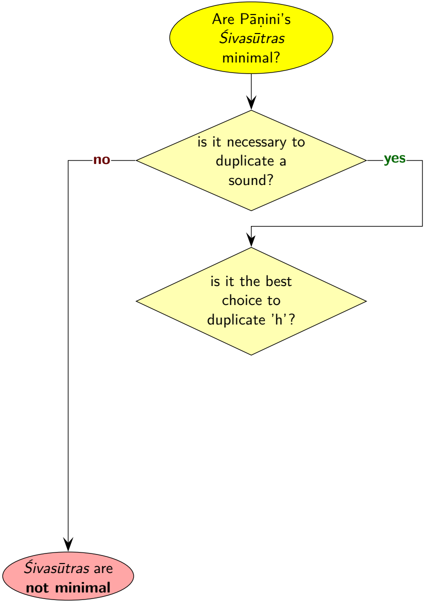
$jhŚ = \{d, ḍ, g, b, j, dh, ḍh, gh, bh, jh\}$ and

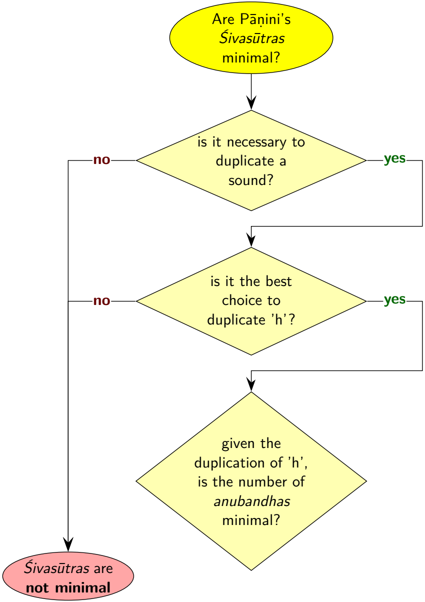
$jhṢ = \{dh, ḍh, gh, bh, jh\}$

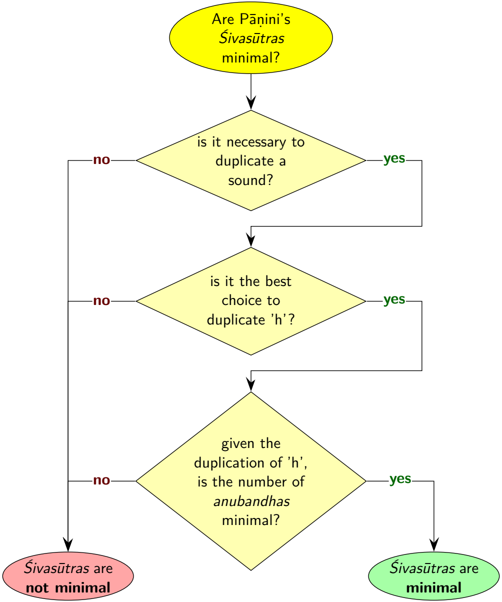
$\Rightarrow h < s, ṣ, ś < p, k, t, ṭ, c, th, ṭh, ch, ph, kh, d < ḍ, g, b, j <$
$dh, ḍh, gh, bh, jh$

Are Pāṇini's *Śivasūtras* minimal?


Are Pāṇini's *Śivasūtras* minimal?
is it necessary to duplicate a sound?


Are Pāṇini's *Śivasūtras* minimal?
is it necessary to duplicate a sound?
no
yes
is it the best choice to duplicate 'h'?
Śivasūtras are **not minimal**

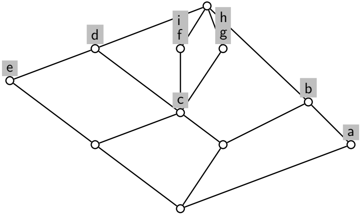

Are Pāṇini's *Śivasūtras* minimal?
is it necessary to duplicate a sound?
no
yes
is it the best choice to duplicate 'h'?
no
yes
given the duplication of 'h', is the number of *anubandhas* minimal?
Śivasūtras are **not minimal**


Are Pāṇini's *Śivasūtras* minimal?
is it necessary to duplicate a sound?
no
yes
is it the best choice to duplicate 'h'?
no
yes
given the duplication of 'h', is the number of *anubandhas* minimal?
no
yes
Śivasūtras are **not minimal**
Śivasūtras are **minimal**

Is it necessary to duplicate a sound?

Main theorem on S-sortability (part 1a)

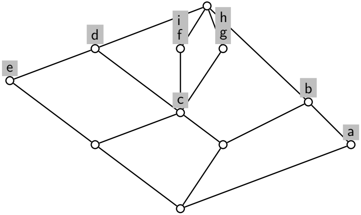
If a set of classes is S-sortable, then its concept lattice is Hasse-planar.

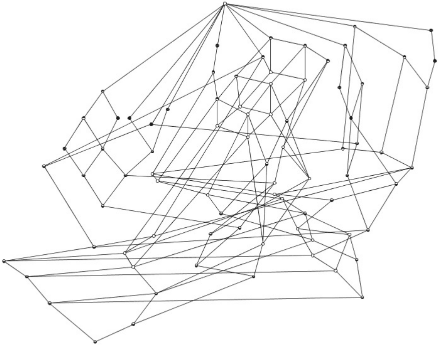


Is it necessary to duplicate a sound?

Main theorem on S-sortability (part 1a)

If a set of classes is S-sortable, then its concept lattice is Hasse-planar.

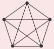


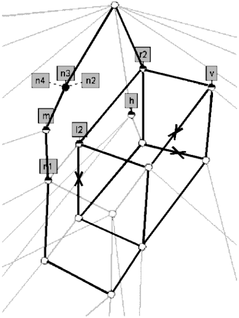


graph of the concept lattice of Pāṇini's *pratyāhāras*

No S-alphabet without duplications for Pāṇini's *pratyāhāras*

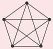
Criterion of Kuratowski

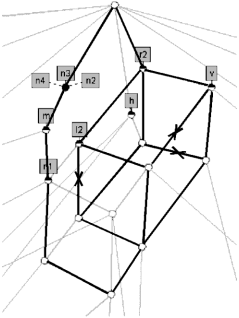
A graph which has the graph  as a minor is not planar.

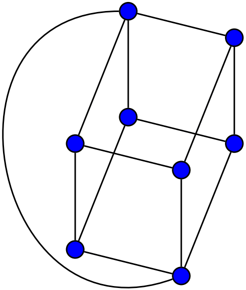


No S-alphabet without duplications for Pāṇini's *pratyāhāras*

Criterion of Kuratowski

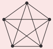
A graph which has the graph  as a minor is not planar.

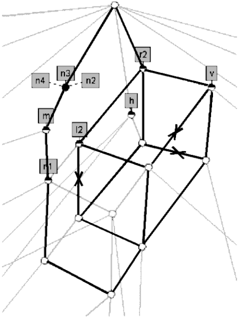


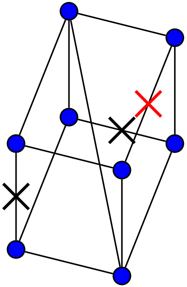


No S-alphabet without duplications for Pāṇini's *pratyāhāras*

Criterion of Kuratowski

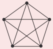
A graph which has the graph  as a minor is not planar.

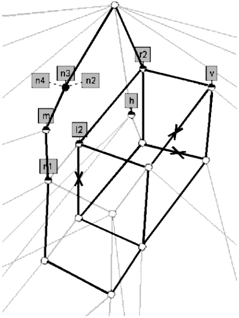


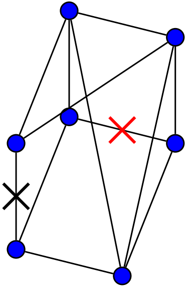


No S-alphabet without duplications for Pāṇini's *pratyāhāras*

Criterion of Kuratowski

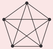
A graph which has the graph  as a minor is not planar.

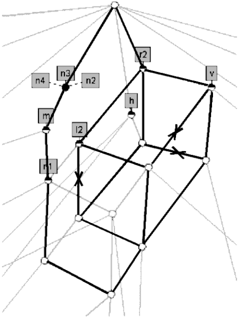


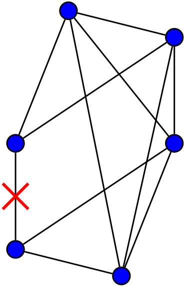


No S-alphabet without duplications for Pāṇini's *pratyāhāras*

Criterion of Kuratowski

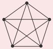
A graph which has the graph  as a minor is not planar.

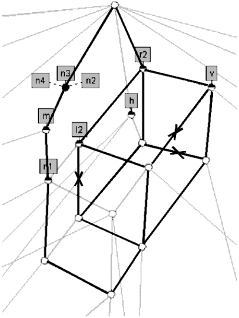


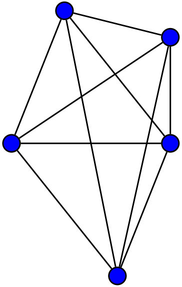


No S-alphabet without duplications for Pāṇini's *pratyāhāras*

Criterion of Kuratowski

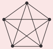
A graph which has the graph  as a minor is not planar.

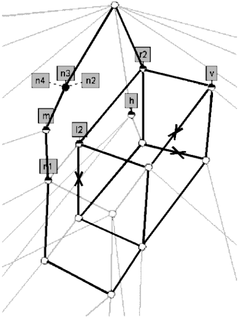


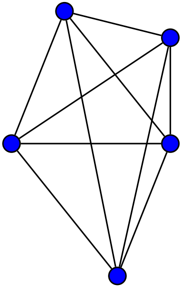


No S-alphabet without duplications for Pāṇini's *pratyāhāras*

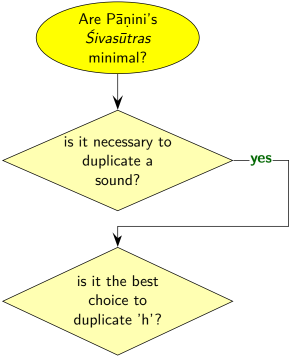
Criterion of Kuratowski

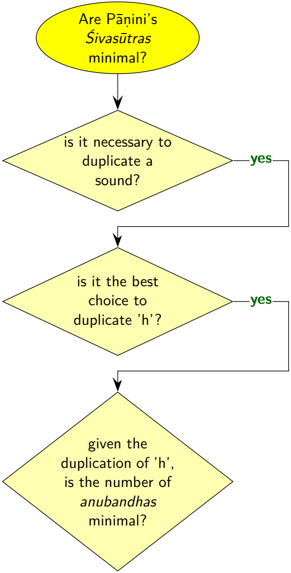
A graph which has the graph  as a minor is not planar.



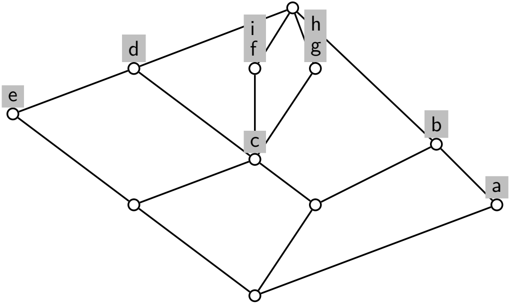


There is no S-alphabet for the set of classes given by Pāṇini's *pratyāhāras* without duplicated elements!


Are Pāṇini's *Śivasūtras* minimal?
is it necessary to duplicate a sound?
yes
is it the best choice to duplicate 'h'?


Are Pāṇini's *Śivasūtras* minimal?
is it necessary to duplicate a sound?
yes
is it the best choice to duplicate 'h'?
yes
given the duplication of 'h', is the number of *anubandhas* minimal?

S-alphabets with a minimal number of markers

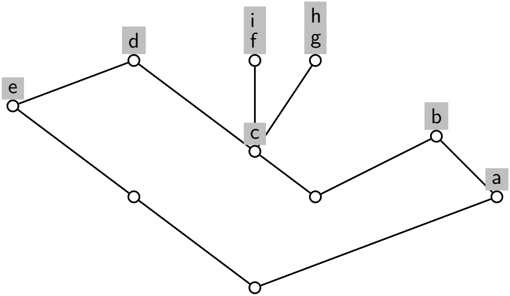


procedure

Start with the empty sequence and choose a walk through the S-graph:

- While moving upwards do nothing.
- While moving downwards along an edge add a new marker to the sequence unless its last element is already a marker.
- If a labeled node is reached, add the labels in arbitrary order to the sequence, unless it has been added before.

S-alphabets with a minimal number of markers

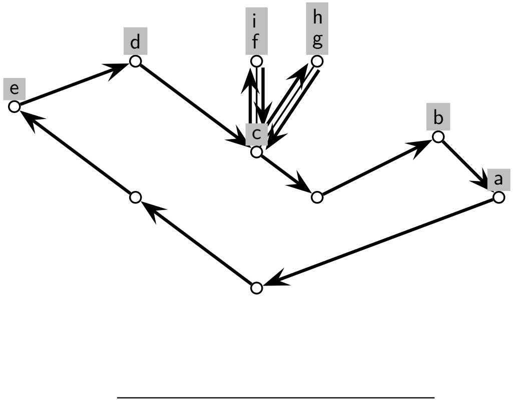


procedure

Start with the empty sequence and choose a walk through the S-graph:

- While moving upwards do nothing.
- While moving downwards along an edge add a new marker to the sequence unless its last element is already a marker.
- If a labeled node is reached, add the labels in arbitrary order to the sequence, unless it has been added before.

S-alphabets with a minimal number of markers

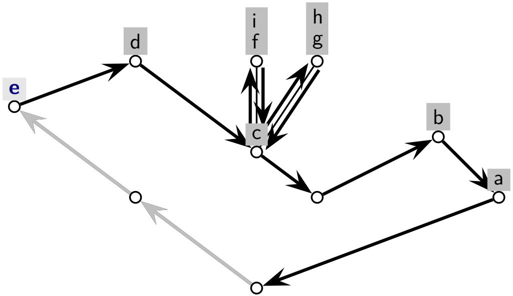


procedure

Start with the empty sequence and choose a walk through the S-graph:

- While moving upwards do nothing.
- While moving downwards along an edge add a new marker to the sequence unless its last element is already a marker.
- If a sound is reached, add the sound to the sequence, unless it has been added before.

S-alphabets with a minimal number of markers

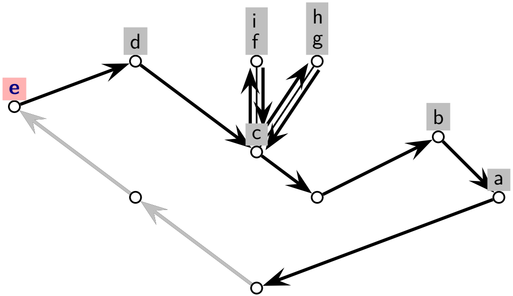


procedure

Start with the empty sequence and choose a walk through the S-graph:

- While moving upwards do nothing.
- While moving downwards along an edge add a new marker to the sequence unless its last element is already a marker.
- If a sound is reached, add the sound to the sequence, unless it has been added before.

S-alphabets with a minimal number of markers



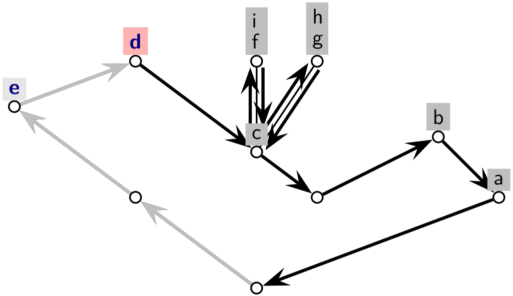
e

procedure

Start with the empty sequence and choose a walk through the S-graph:

- While moving upwards do nothing.
- While moving downwards along an edge add a new marker to the sequence unless its last element is already a marker.
- If a sound is reached, add the sound to the sequence, unless it has been added before.

S-alphabets with a minimal number of markers



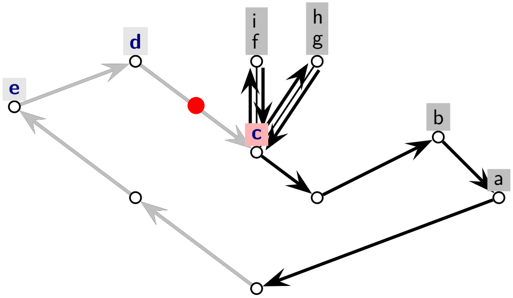
ed

procedure

Start with the empty sequence and choose a walk through the S-graph:

- While moving upwards do nothing.
- While moving downwards along an edge add a new marker to the sequence unless its last element is already a marker.
- If a sound is reached, add the sound to the sequence, unless it has been added before.

S-alphabets with a minimal number of markers



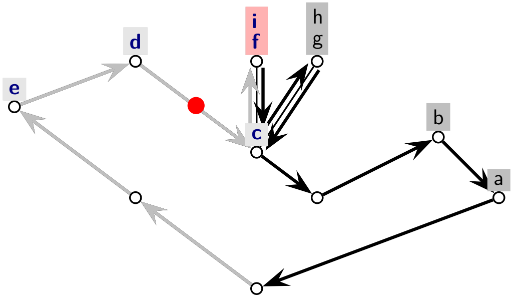
edM_1c

procedure

Start with the empty sequence and choose a walk through the S-graph:

- While moving upwards do nothing.
- While moving downwards along an edge add a new marker to the sequence unless its last element is already a marker.
- If a sound is reached, add the sound to the sequence, unless it has been added before.

S-alphabets with a minimal number of markers



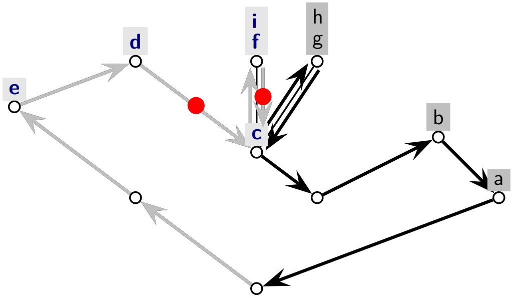
$ed\,M_1\,cfi$

procedure

Start with the empty sequence and choose a walk through the S-graph:

- While moving upwards do nothing.
- While moving downwards along an edge add a new marker to the sequence unless its last element is already a marker.
- If a sound is reached, add the sound to the sequence, unless it has been added before.

S-alphabets with a minimal number of markers



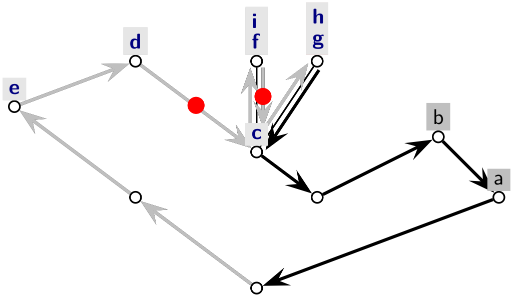
$ed\,M_1\,cfi\,M_2$

procedure

Start with the empty sequence and choose a walk through the S-graph:

- While moving upwards do nothing.
- While moving downwards along an edge add a new marker to the sequence unless its last element is already a marker.
- If a sound is reached, add the sound to the sequence, unless it has been added before.

S-alphabets with a minimal number of markers



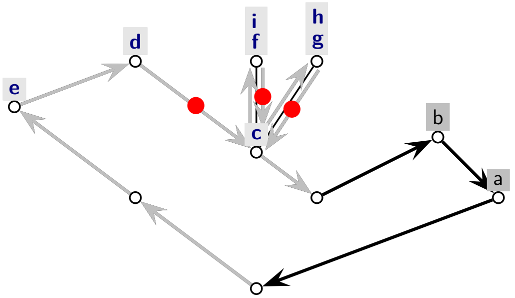
$ed\,M_1\,c\,fi\,M_2\,gh$

procedure

Start with the empty sequence and choose a walk through the S-graph:

- While moving upwards do nothing.
- While moving downwards along an edge add a new marker to the sequence unless its last element is already a marker.
- If a sound is reached, add the sound to the sequence, unless it has been added before.

S-alphabets with a minimal number of markers



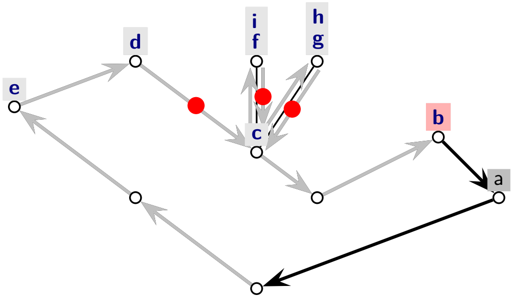
$edM_1cfiM_2ghM_3$

procedure

Start with the empty sequence and choose a walk through the S-graph:

- While moving upwards do nothing.
- While moving downwards along an edge add a new marker to the sequence unless its last element is already a marker.
- If a sound is reached, add the sound to the sequence, unless it has been added before.

S-alphabets with a minimal number of markers



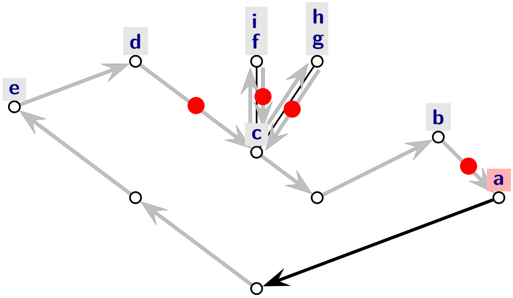
$ed\,M_1\,c\,f\,i\,M_2\,g\,h\,M_3\,b$

procedure

Start with the empty sequence and choose a walk through the S-graph:

- While moving upwards do nothing.
- While moving downwards along an edge add a new marker to the sequence unless its last element is already a marker.
- If a sound is reached, add the sound to the sequence, unless it has been added before.

S-alphabets with a minimal number of markers



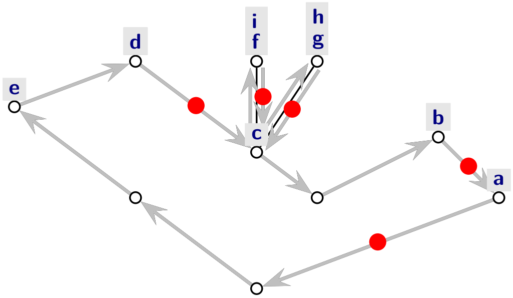
$ed\,M_1\,cfi\,M_2\,gh\,M_3\,b\,M_4\,a$

procedure

Start with the empty sequence and choose a walk through the S-graph:

- While moving upwards do nothing.
- While moving downwards along an edge add a new marker to the sequence unless its last element is already a marker.
- If a sound is reached, add the sound to the sequence, unless it has been added before.

S-alphabets with a minimal number of markers



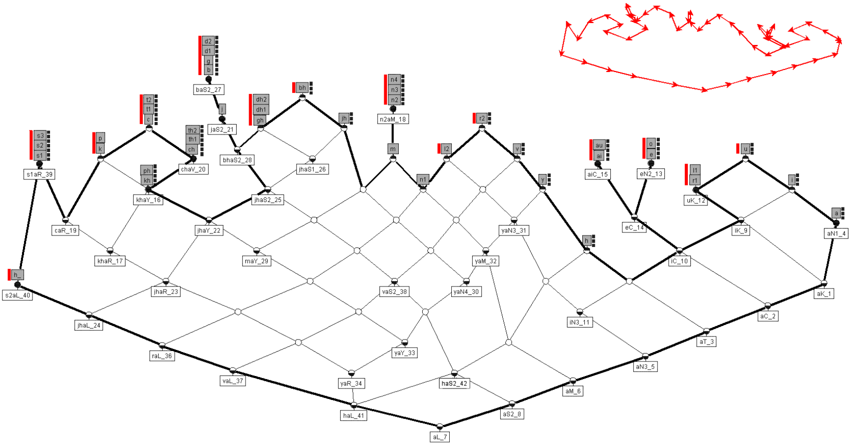
$ed\,M_1\,cfi\,M_2\,gh\,M_3\,b\,M_4\,a\,M_5$

procedure

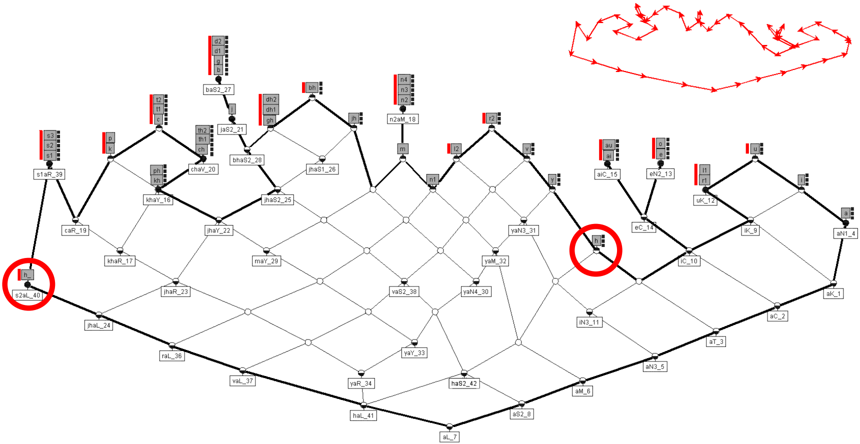
Start with the empty sequence and choose a walk through the S-graph:

- While moving upwards do nothing.
- While moving downwards along an edge add a new marker to the sequence unless its last element is already a marker.
- If a sound is reached, add the sound to the sequence, unless it has been added before.

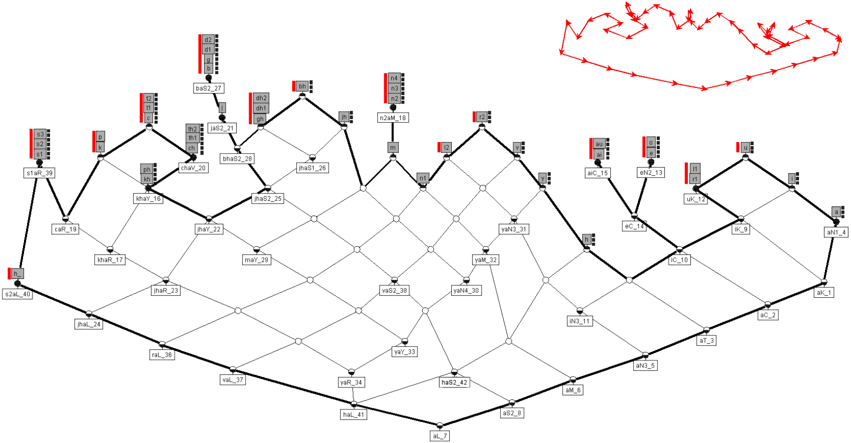
Concept lattice of Pāṇini's *pratyāhāras* with duplicated *h*



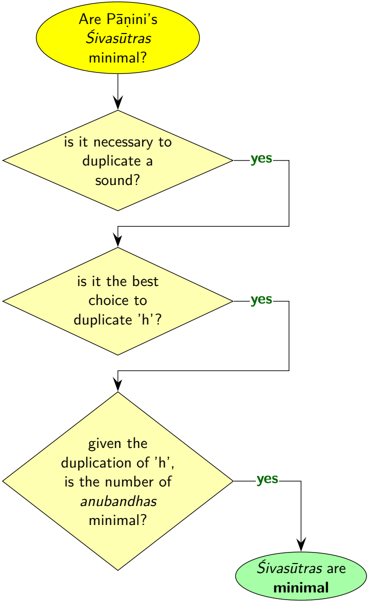
Concept lattice of Pāṇini's *pratyāhāras* with duplicated *h*



Concept lattice of Pāṇini's *pratyāhāras* with duplicated *h*

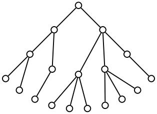


With the *Śivasūtras* Pāṇini has chosen one out of nearly 12 million minimal S-alphabets!

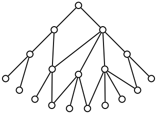

Are Pāṇini's *Śivasūtras* minimal?
is it necessary to duplicate a sound?
yes
is it the best choice to duplicate 'h'?
yes
given the duplication of 'h', is the number of *anubandhas* minimal?
yes
Śivasūtras are **minimal**

Transfer

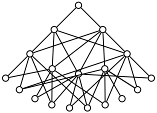
- For physical objects ‚duplicating‘ means ‚adding copies‘
- Adding copies is annoying but often not impossible
- Ordering objects in an S-order may
  - improve user-friendliness
  - save time
  - save space
  - simplify visual representations of classifications



tree



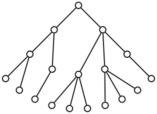
S-sortable



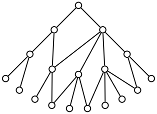
general hierarchy

Transfer

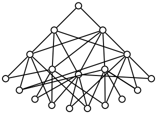
- For physical objects ‚duplicating‘ means ‚adding copies‘
- Adding copies is annoying but often not impossible
- Ordering objects in an S-order may
    - improve user-friendliness
    - save time
    - save space
    - simplify visual representations of classifications



tree



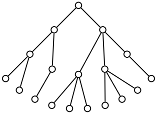
S-sortable



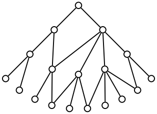
general hierarchy

Transfer

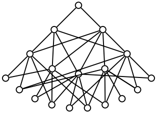
- For physical objects ,duplicating‘ means ,adding copies‘
- Adding copies is annoying but often not impossible
- Ordering objects in an S-order may
  - improve user-friendliness
  - save time
  - save space
  - simplify visual representations of classifications



tree



S-sortable



general hierarchy

Transfer

Objects in libraries, ware-houses, and stores are only *nearly* linearly arranged:

⇒ Second (and third) dimension can be used in order to avoid duplications

Open problems

What explains the actual structure of the *Śivasūtras*?

- **principle of homorganic continuity** (Staal, 1962)
- **principle of historic continuity** (Cardona, 1969)
- **principle of economy** and **logic of the special case and the general case** (Kiparsky 1991) or **Pāṇini's razor** (Kiparsky 2007)

The presented approach cannot give an answer to this question

The story is much more intricate

- We have neither shown that Pāṇini's technique for the representation of sound classes is optimal
- nor that he has used his technique in an optimal way.

Open problems

What explains the actual structure of the *Śivasūtras*?

- **principle of homorganic continuity** (Staal, 1962)
- **principle of historic continuity** (Cardona, 1969)
- **principle of economy** and **logic of the special case and the general case** (Kiparsky 1991) or **Pāṇini's razor** (Kiparsky 2007)

The presented approach cannot give an answer to this question

The story is much more intricate

- We have neither shown that Pāṇini's technique for the representation of sound classes is optimal
- nor that he has used his technique in an optimal way.

Open problems

What explains the actual structure of the *Śivasūtras*?

- **principle of homorganic continuity** (Staal, 1962)
- **principle of historic continuity** (Cardona, 1969)
- **principle of economy** and **logic of the special case and the general case** (Kiparsky 1991) or **Pāṇini's razor** (Kiparsky 2007)

The presented approach cannot give an answer to this question

The story is much more intricate

- We have neither shown that Pāṇini's technique for the representation of sound classes is optimal
- nor that he has used his technique in an optimal way.

Literature

- Cardona, G. (1969), Studies in Indian grammarians I: The method of description reflected in the Śiva-Sūtras. In: Transactions of the American Philosophical Society, 59(1):3–48.
- Kiparsky, P. (1991), Economy and the construction of the Śivasūtras. In: M. M. Deshpande & S. Bhate (eds.), *Pāṇinian Studies*, Michigan: Ann Arbor.
- Petersen, W. (2008), Zur Minimalität von Pāṇinis *Śivasūtras* – Eine Untersuchung mit Mitteln der Formalen Begriffsanalyse. PhD thesis, university of Düsseldorf.
- Petersen, W. (2009), On the Construction of Sivasutra-Alphabets. In: A. Kulkarni and G. Huet (eds.): *Sanskrit Computational Linguistics.* LNCS 5406, Springer.
- Staal, F. (1962), A Method of Linguistic Description. Language **38**, 1-10.

Origin of Pictures

- libraries (left): `http://www.meduniwien.ac.at/medizinischepsychologie/bibliothek.htm`
- libraries (middle): `http://www.math-nat.de/aktuelles/allgemein.htm`
- libraries (right): `http://www.geschichte.mpg.de/deutsch/bibliothek.html`
- warehouses: `http://www.metrogroup.de/servlet/PB/menu/1114920_l1/index.html`
- stores: `http://www.einkaufsparadies-schmidt.de/01bilder01/`