Complexity of Natural Languages
Mildly-context sensitivity
T1 languages

Introduction to Formal Language Theory — day 4

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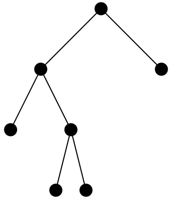
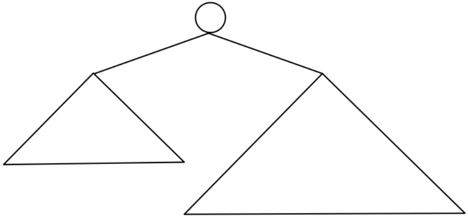
NASSLLI 2014

Outline

binary trees





Proposition

If T is an arbitrary binary tree with at least 2^k leafs, then height$(T) \geq k$.

Proof by induction on k. The proposition is true for $k = 0$. Given the proposition is true for some fixed k, let T be a tree with $\geq 2^{k+1}$ leafs. T has two subtrees of which at least one has 2^k leafs. Thus the height of T is $\geq 2^{k+1}$.

Corollary

If a context-free grammar is in CNF, then the height of a derivation tree of a word of length $\geq 2^k$, then height(T) is greater than k (note that the last derivation step is always a unary one).

Pumping lemma for context-free languages

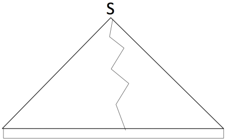
Lemma (Pumping Lemma)

For each context-free language L there exists a $n \in \mathbb{N}$ such that for any $z \in L$: if $|z| \geq n$, then z may be written as $z = uvwxy$ with

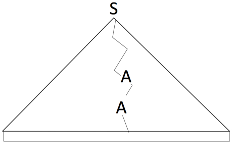
- $u, v, w, x, y \in T^*$,
- $|vwx| \leq p$,
- $vx \neq \epsilon$ *and*
- $uv^iwx^iy \in L$ *for any* $i \geq 0$.

Pumping lemma: proof sketch

Let $k = |N|$ and $n = 2^k$. Be $z \in L$ with $|z| \geq n$.

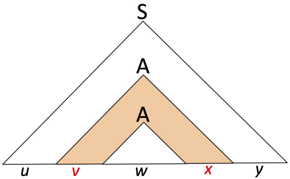


Because of $|z| \geq 2^k$ there exists a path in the binary part of the derivation tree of z of length $\geq k$.



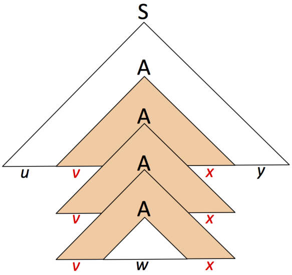
At least one non-terminal symbol occurs twice on the path.
Starting from the bottom of the path, let A be the first non-terminal occurring twice.

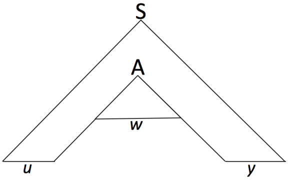
Pumping Lemma: proof sketch



$|vwx| \leq n$ (A is chosen such that no non-terminal occurs twice in the trees spanned by the upper of the two A's)
$vx \neq \epsilon$ (a binary rule $A \rightarrow BC$ must have been applied to the upper A).

Pumping Lemma: proof sketch





$uv^i wx^i y \in L$ for any $i \geq 0$.

Pumping Lemma: application

The language $L(a^k b^m c^k d^m)$ is not context-free

- Assume that $L(a^k b^m c^k d^m)$ is context-free then there is a $n \in \mathbb{N}$ as specified by the Pumping Lemma.
- Choose $z = a^n b^n c^n d^n$, and $z = uvwxy$ in accordance with the Pumping Lemma.
- Because of $vwx \leq n$ the string *vwx* consists either of only *a*'s, of *a* and *b*'s, only of *b*'s, of *b* and *c*'s, only of *c*'s,....
- It follows that the pumped word uv^2wx^2y cannot be in L.
- That contradicts the assumption that L is context-free.

Closure properties of context-free languages

	Type3	Type2	Type1	Type0
union	+	+	+	+
intersection	+	-	+	+
complement	+	-	+	-
concatenation	+	+	+	+
Kleene's star	+	+	+	+
intersection with a regular language	+	+	+	+

union: $G = (N_1 \uplus N_2 \cup \{S\}, T_1 \cup T_2, S, P)$ with
$P = P_1 \cup_{\uplus} P_2 \cup \{S \to S_1, S \to S_2\}$

intersection: $L_1 = \{a^n b^n a^k\}$, $L_2 = \{a^n b^k a^k\}$, but $L_1 \cap L_2 = \{a^n b^n a^n\}$

complement: *de Morgan*

concatenation: $G = (N_1 \uplus N_2 \cup \{S\}, T_1 \cup T_2, S, P)$ with $P = P_1 \cup_{\uplus} P_2 \cup \{S \to S_1 S_2\}$

Kleene's star: $G = (N_1 \cup \{S\}, T_1, S, P)$ with $P = P_1 \cup \{S \to S_1 S, S \to \epsilon\}$

Are natural languages context-free?

- a long time debate about the context-freeness of natural languages

  Chomsky (1957) : “Of course there are languages (in our general sense) that cannot be described in terms of phrase structure, but I do not know whether or not English is itself literally outside the range of such analysis.”

- several wrong arguments, e.g.:

  Bresnan (1987): : “in many cases the number of a verb agrees with that of a noun phrase at some distance from it ... this type of syntactic dependency can extend as memory or patience permits ... the distant type of agreement ... cannot be adequately described even by context-sensitive phrase-structure rules, for the possible context is not correctly describable as a finite string of phrases."

- right proof techniques: pumping lemma and closure properties
- a non context-free phenomenon: **cross-serial dependencies** in Schwyzerdütsch (Schieber 1985)

Are natural languages context-free?

- German: **nested dependency** (subordinate clauses)

(1) er die Kinder dem Hans das Haus streichen helfen ließ.
he the children the Hans the house paint help let.
'he he let the children to help Hans to paint the house.'

$n1 \quad n2 \quad n3 \quad v3 \quad v2 \quad v1$

Are natural languages context-free?

- Schwyzerdütsch: **cross-serial dependency**

(2)

mer	d'chind	em Hans	es huus	lönd	hälfe	aastriiche.
we	children.acc	the Hans.dat	the house.acc	let	help	paint.

'that we let the children to help Hans to paint the house.'

$n1 \quad n2 \quad n3 \quad v1 \quad v2 \quad v3$

(3)

*mer	d'chind	de Hans	es huus	lönd	hälfe	aastriiche.
we	children.acc	the Hans.acc	the house.acc	let	help	paint.

Proof by Schieber

- Jan säit das mer d'chind em Hans es huus lönd hälfe aastriiche.
- homomorphism f:

f(d'chind) $= a$	f(em Hans) $= b$	f(laa) $= c$
f(hlfe) $= d$	f(aastriiche) $= y$	f(es huus haend wele) $= x$
f(Jan sit das mer) $= w$	$f(s) = z$ *otherwise*	

- $f(\text{Schwyzerdütsch}) \cap wa^*b^*xc^*d^*y = wa^mb^nxc^md^ny$
  - CF languages are closed under intersection with regular languages
  - $wa^*b^*xc^*d^*y$ is regular
  - by Pumping Lemma: $wa^mb^nxc^md^ny$ is not regular
- $\Rightarrow$ Schwyzerdütsch is not context-free

Duplication

- duplication (in morphology): Bambara (spoken in Mali)
  - wulu *'dog'*
    wulu-lela *'dog watcher'*
    wulu-lela-nyinila *'dog watcher hunter'*
    wulu-o-wulu *'whatever dog'*
    wulu-lela-o-wulu-lela *'whatever dog watcher'*
    wulu-lela-nyinila-o-wulu-lela-nyinila *'whatever dog watcher hunter'*
- structure of the form $x = yy \Rightarrow$ not context-free

Dutch cross dependencies

- cross dependencies in Dutch

(4)

dat	Jan	Piet	de	kinderen	zag	helpen	zwemmen.
that	Jan	Piet	the	children	saw	help	swim

'that Jan saw Piet helping the children to swim.'

  - no case marking → string can be generated by a CFG

    $n1 \quad n2 \quad n3 \quad v3 \quad v2 \quad v1$

  - however the linguistic dependencies are not preserved ⇒ structure (predicate-argument relations)
  - weak generative capacity: preserve the string language
  - strong generative capacity: preserve the structure

Mildly context sensitive grammars

- for natural languages context-free grammars are just not ‘enough’
  - not context-free structures / languages:

(1) $\{a^n b^n c^n \mid n \geq 0\}$ (multiple agreement)
(2) $\{a^n b^m c^n d^m \mid m, n \geq 0\}$ (cross-serial dependencies)
(3) $\{ww \mid w \in \{a, b\}^*\}$ (duplication)

- for natural languages we need grammars, that are somewhat richer than context-free grammars, but more restricted than context-sensitive grammars
- $\Rightarrow$ natural languages are almost context-free
- “mildly context sensitive" (Joshi, 1985)
- $RL \subset CFL \subset MCSL \subset CSL \subset RE$

Mildly context sensitive languages

Definition: Mildly context-sensitive language (Joshi, 1985)

1. A set $\mathcal{L}$ of languages is mildly context-sensitive iff
   a. $\mathcal{L}$ contains all context-free languages
   b. $\mathcal{L}$ can describe **cross-serial dependencies**: there is an $n \geq 2$ such that $\{w^k \mid w \in (V_T)^*\} \in L$ for all $k \geq n$
   c. the languages in $\mathcal{L}$ are **polynomially parseable**, i.e., $L \subset$ PTIME
   d. the languages in $\mathcal{L}$ have the constant growth property
2. A formalism F is mildly context-sensitive iff the set $\{L \mid L = L(G) \text{ for some } G \in F\}$ is mildly context-sensitive.

Mildly context sensitive grammars

- mildly context-sensitive grammar formalisms
  - Linear Indexed Grammars (LIGs)
  - Head Grammars (HGs)
  - Tree Adjoining Grammars (TAGs)
  - Multicomponent TAGs (MCTAGs)
  - Combinatory Categorial Grammars (CCGs)
  - Linear Context-Free Rewriting Systems (LCFRSs)
- TAGs, CCGs, LIGs and HGs are weakly equivalent
- MCTAGs and LCFRSs subsume TAGs, CCGs, LIGs and HGs

Context-sensitive languages

Definition

A grammar (N, T, S, R) is **Type1** or **context-sensitive** iff all rules are of the form:

$\gamma A \delta \rightarrow \gamma \beta \delta$ with $\gamma, \delta, \beta \in (N \cup T)^*, A \in N$ and $\beta \neq \epsilon$;

With the exception that $S \rightarrow \epsilon$ is allowed if S does not occur in any rule's right-hand side.

A language generated by a T1 grammar is said to be a **context-sensitive** or **Type1-language**.

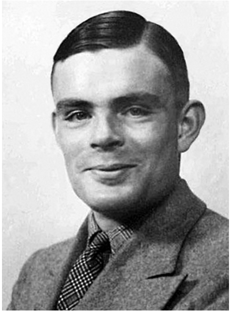
- γ en δ can be empty, but β cannot be the empty string; $\beta \neq \epsilon$ (!)
  $\rightsquigarrow$ 'non-shrinking' context-sensitive scheme

Example: CS grammar

- consider the language $a^n b^n c^n$
- a context-sensitive grammar generating this language is:
  - $G = (T, N, S, R)$ with
    $T = \{a, b, c\}$
    $N = \{S, A, B, C, T\}$
    $R = \{S \rightarrow \epsilon$, S→ T,
    T→ aBC, T→ aTBC,
    (recursively generating $a^n(BC)^n$)
    CB → CX, CX → BX, BX → BC,
    (swapping two non-terminals: $CB \rightarrow BC$)
    aB → ab, bB → bb, bC → bc, cC → cc}
    (from $a^n B^n C^n$ to $a^n b^n c^n$)
  - S ⇒ T ⇒* aaaaBCBCBCBC ⇒* aaaaBCBCBCBC ⇒* aaaaBBCCBCBC ⇒* aaaaBBCBCCBC ⇒* aaaaBBBBCCCC ⇒ aaaabBBBCCCC ⇒ aaaabbBBCCCC ⇒* aaaabbbbCCCC ⇒ aaaabbbbcCCC ⇒ aaaabbbbccCC ⇒* aaaabbbbcccc

Turing machine

- unrestricted grammars generate Type 0 languages
- Turing machines recognize (or generate) Type 0 languages



Alan Turing

(1912 – 1952)

Turing machine: an abstract 'computer'

"Computing is normally done by writing certain symbols on paper. We may suppose this paper is divided into squares like a child's arithmetic book. [. . .]
I think that it is agreed that the two-dimensional character of paper is no essential of computation. I assume then that the computation is carried out on one-dimensional paper, i.e. on tape divided into squares."

[Alan Turing: On computable numbers with an application to the Entscheidungsproblem. In: *Proceedings of the London Mathematical Society*, 2, 1936.]

Turing machine

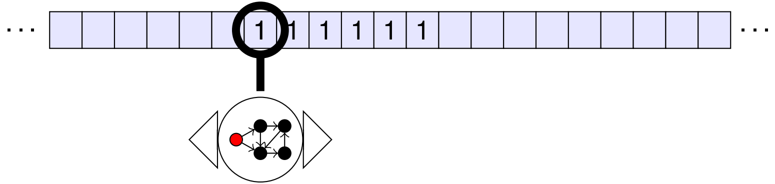
Definition

A deterministic ***Turing machine*** *is a tuple* $(Q, \Sigma, \Gamma, \delta, q_0, \square, F)$ *with:*

- *Q is a finite, non-empty set of* ***states***
- *$\Sigma \subset \Gamma$ is the set of the* ***input symbols***
- *Γ is the finite, non-empty set of the* ***tape symbols***
- *$\delta : Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$ is the partial* ***transition function*** *with L for left and R for right move.*
- *$q_0 \in Q$ is the* ***initial state***
- *$\square \in \Gamma \setminus \Sigma$ is the* ***blank symbol***
- *$F \subseteq Q$ is the set of* ***accepting states***

Note: the transition function is partial, i.e. for some state tape-symbol pairs $\delta(q, a)$ is undefined.

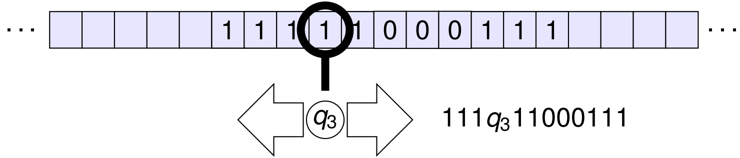
An image of a Turing machine



Start conventions

- The tape of the TM contains the input string. All other tape positions are filled by the blank symbol □.
- The (read-write) head of the TM is placed above the left-most input symbol.
- The TM is in the start state.

Configurations



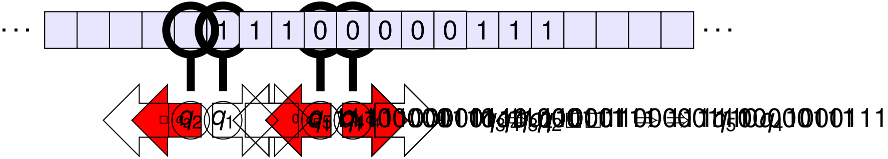
$111q_311000111$

A **configuration** of a TM is a string $\alpha q \beta$, where

- α is the string of symbols to the left of the head starting with the left-most non-blank symbol on the tape
- q is the state the TM is in.
- β is the rest of the string ending with the right-most non-blank symbol on the tape.
- the read-write head is currently scanning the first symbol of β.

$\alpha\beta$ must be finite for any configuration of a TM as every configuration of a TM is reached after a finite number of steps (i.e., the head can only be moved a finite number of positions to the right or to the left from the starting position).

Transitions



Transitions

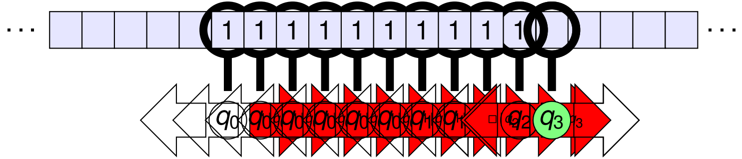
- $\delta(q, a) = (q', b, R|L)$ specifies that if the TM is in state q and reads an a it can change to state q', write b, and move either one position right (R) or left (L).
- For a right-move transition $\delta(q, a) = (q', b, R)$ we get: $\alpha qa\beta \Rightarrow \alpha bq'\beta$.
- For a left-move transition $\delta(q, a) = (q', b, L)$ we get: $\alpha cqa\beta \Rightarrow \alpha q' cb$

$\Rightarrow^*$ is used as before for the closure of $\Rightarrow$

Example: Turing machine for addition

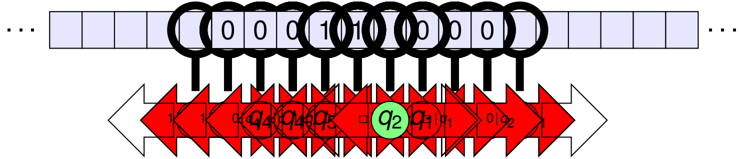
- Take the following Turing machine: $M = (\{q_0, q_1, q_2\}, \{1\}, \{1\}, q_0, \delta, \{q_3\})$
  - states: q_0, q_1, q_2
  - alphabet and input alphabet: $\{1\}$
  - final state: q_3
  - transitions: $\delta = \{(q_0, 1) \to (q_0, 1, R), (q_0, \Box) \to (q_1, 1, R),$
    $(q_1, 1) \to (q_1, 1, R), (q_1, \Box) \to (q_2, \Box, L),$
    $(q_2, 1) \to (q_3, \Box, R)\}$

Example: Turing machine for addition



	1	□
q_0	$(q_0, 1, R)$	$(q_1, 1, R)$
q_1	$(q_1, 1, R)$	$(q_2, \square, L)$
q_2	$(q_3, \square, R)$	
$\underline{q_3}$		

Turing machine for subtraction



	1	□	0
q_0	$(q_0, 1, R)$	$(q_1, \square, R)$	
q_1	$(q_1, 1, R)$	$(q_2, 0, L)$	$(q_2, \square, L)$
q_2			$(q_3, 0, L)$
q_3	$(q_3, 1, L)$	$(q_4, \square, L)$	
q_4	$(q_4, 1, L)$	$(q_5, \square, R)$	$(q_5, 0, R)$
q_5			$(q_0, 0, R)$

Language accepted by a TM

Acceptance by final state

A turing machine M **accepts** the language $L(M)$ by final state:
$L(M) = \{w \mid q_0 w \Rightarrow^* C$ where C is a configuration with a final state$\}$

Acceptance by halting

A turing machine M **accepts** the language $H(M)$ by halting:
$H(M) = \{w \mid q_0 w \Rightarrow^* C$ where C is a configuration without possible moves$\}$

Equivalence of acceptance by finite state and by halting

- If $L = L(M)$, then there exists a TM M' with $L = H(M')$.
  (remove all moves from the final state)
- If $L = H(M)$, then there exists a TM M" with $L = L(M'')$.
  (transition to a new final state from all pairs for which $\delta(q, a)$ is undefined).

Turing machines accept the recursively enumerable languages (RE).

Turing-computable functions

TMs can be seen as acceptors (accepting languages) or as computers (computing functions).

A partial function $f : \Sigma^* \to \Sigma^*$ is Turing-computable if there exists a TM $(Q, \Sigma, \Gamma, \delta, q_0, \Box, F)$ such that:

$$f(w) = v \text{ if and only if } (\epsilon, q_0, w) \Rightarrow^* (\epsilon, q_f, f(w)) \text{ with } q_f \in F.$$

Church's thesis

Every effective computation can be carried out by a Turing machine.
Everything that is in some intuitive way computable is Turing-computable.

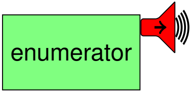
Enumerations

enumerable

A language $L \subseteq \Sigma^*$ is enumerable, if $L = \emptyset$ or there exists a total function $f : \mathbb{N} \rightarrow \Sigma^*$ such that $L = \{f(n) | n \in \mathbb{N}\}$.

recursively enumerable

A language $L \subseteq \Sigma^*$ is recursively enumerable, if $L = \emptyset$ or there exists a total computable function $f : \mathbb{N} \rightarrow \Sigma^*$ such that $L = \{f(n) | n \in \mathbb{N}\}$



$\ldots, w_{14}, w_{13}, w_{12}, w_{11}, w_{10}, w_9, w_8, w_7, w_6, w_5, w_4, w_3, w_2, w_1$

Proposition

Every language accepted by a Turing machine is recursively enumerable.

TM extensions

multi-track TMs

If a language L is accepted by a TM with any finite number of tracks, then there is a TM with one tape which accepts L.

A multi-track TM consists of a finite number of tapes, called tracks; the head scans all tapes at the same position and moves on all tapes in simultaneously (analogue to 1-tape TM with a tape alphabet of vectors).

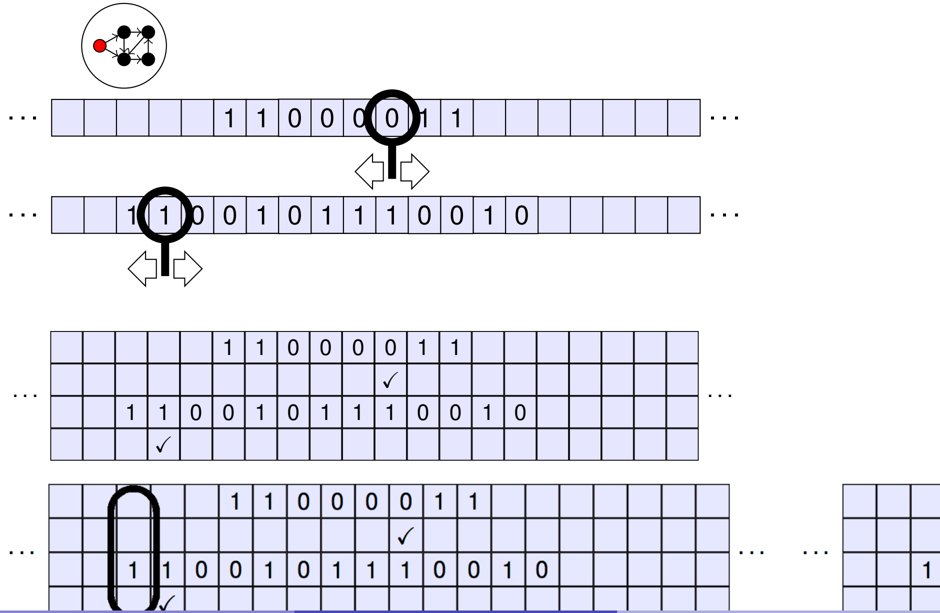
multi-tape TMs

If a language L is accepted by a TM with any finite number of tapes, then there is a TM with one tape which accepts L.

In a multi-tape TM the head can move independently on all tapes.
A 2-tape TM is simulated by a 4-track TM, where

- the 1st track simulates the tape of the 1st TM.
- the 2nd track simulates the position of the head of the 1st TM.
- the 3rd track simulates the tape of the 2nd TM.
- the 4th track simulates the position of the head of the 2nd TM.

multi-tape TM → multi-track TM



Further extensions and restrictions of TMs

Extension: nondeterministic TMs

If a language L is accepted by a nondeterministic Turing machine, then there is a deterministic Turing machine which accepts L.

Restrictions of TMs

- Push-down automata with two stacks have the same expressive power as Turing machines.
- Turing machines with semi-bounded tapes (the tape only grows into one direction) have the same expressive power as Turing machines.

Linearly bounded TMs

Turing machines with a bounded tape the length of which is linearly bounded by the length of the input string are weaker than general Turing machines. They accept languages of Type 1.

Closure properties of recursively enumerable languages

Recursively enumerable languages are closed under

- union (Given two TMs M and M'. For $H(M) \cup H(M')$ construct a 2-tape Turing machine which simulates M and M' on the two tapes.)
- intersection (Similar construction as for union but with $L(M) \cap L(M')$)
- concatenation (Given two TMs M and M'. For $H(M) \frown H(M')$ construct a 2-tape nondeterministic TM which guesses the breakpoint of an input string and then simulates on the first tape M on the first part of the string and on the second tape M' on the second part.)
- Kleene star (Similar to concatenation)

RE is not closed under complement, as we cannot decide whether a running TM will ever halt.