Parsing Beyond Context-Free Grammars: Introduction

Laura Kallmeyer & Tatiana Bladier
Heinrich-Heine-Universität Düsseldorf

Sommersemester 2018

Overview

[Kal10]

CFG and natural languages (1)

A context-free grammar (CFG) is a set of rewriting rules that tell us how to replace a non-terminal by a sequence of non-terminal and terminal symbols.

Example:

S → a S b   S → ab

The string language generated by this grammar is $\{a^n b^n \mid n \geq 1\}$.

CFG and natural languages (2)

Sample CFG $G_{telescope}$:

S	$\rightarrow$	NP VP	NP	$\rightarrow$	D N
VP	$\rightarrow$	VP PP | V NP	N	$\rightarrow$	N PP
PP	$\rightarrow$	P NP			
N	$\rightarrow$	man | girl | telescope	D	$\rightarrow$	the
N	$\rightarrow$	John	P	$\rightarrow$	with
V	$\rightarrow$	saw			

CFG and natural languages (3)

Context-free languages (CFLs)

- can be recognized in polynomial time ($\mathcal{O}(n^3)$);
- are accepted by push-down automata;
- have nice closure properties (e.g., closure under homomorphisms, intersection with regular languages . . .);
- satisfy a pumping lemma;
- can describe nested dependencies ($\{ww^R \mid w \in T^*\}$).

[HU79]

CFG and natural languages (4)

Question: Is CFG powerful enough to describe all natural language phenomena?

Answer: No. There are constructions in natural languages that cannot be adequately described with a context-free grammar.

Example: cross-serial dependencies in Dutch and in Swiss German.

Dutch:

(1)	... dat	Wim	Jan	Marie	de kinderen	zag	helpen	leren	zwemmen
	... that	Wim	Jan	Marie	the children	saw	help	teach	swim

' ... that Wim saw Jan help Marie teach the children to swim'

CFG and natural languages (5)

Swiss German:

(2) ... das mer em Hans es huus hälfed aastriiche
... that we Hans_{Dat} house_{Acc} helped paint
' ... that we helped Hans paint the house'

(3) ... das mer d'chind em Hans es huus lönd hälfe
... that we the children_{Acc} Hans_{Dat} house_{Acc} let help
aastriiche
paint
' ... that we let the children help Hans paint the house'

Swiss German uses case marking and displays cross-serial dependencies.

[Shi85] shows that Swiss German is not context-free.

CFG and natural languages (6)

If closure under homomorphisms and intersection with regular languages is given, the following holds:

A formalism that can generate cross-serial dependencies can also generate the copy language $\{ww \mid w \in \{a, b\}^*\}$.

The copy language is not context-free.

Therefore we are interested in extensions of CFG in order to describe all natural language phenomena.

CFG and natural languages (7)

Idea [Jos85]: characterize the amount of context-sensitivity necessary for natural languages.

Mildly context-sensitive formalisms have the following properties:

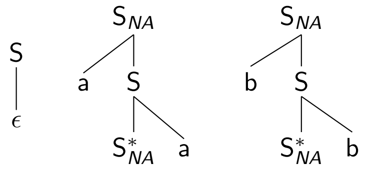
1. They generate (at least) all CFLs.
2. They can describe a limited amount of cross-serial dependencies.
   In other words, there is a $n \geq 2$ up to which the formalism can generate all string languages $\{w^n \mid w \in T^*\}$.
3. They are polynomially parsable.
4. Their string languages are of constant growth.
   In other words, the length of the words generated by the grammar grows in a linear way, e.g., $\{a^{2^n} \mid n \geq 0\}$ does not have that property.

Polynomial extensions of CFG (1)

Tree Adjoining Grammars (TAG), [JLT75, JS97]:

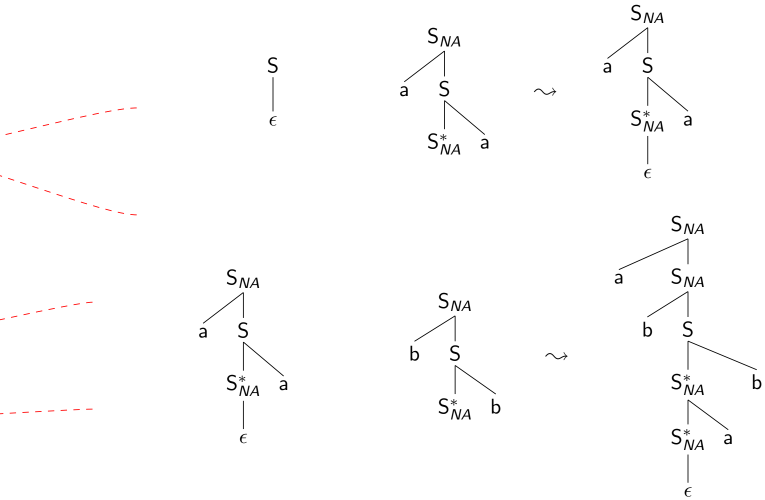
- Tree-rewriting grammar.
- Extension of CFG that allows to replace not only leaves but also internal nodes with new trees.
- Can generate the copy language.

Example: TAG for the copy language



Polynomial extensions of CFG (2)

Example: TAG derivation of *abab*:



Polynomial extensions of CFG (3)

Linear Context-free rewriting systems (LCFRS) and the equivalent Multiple Context-Free Grammars (MCFG), [VSWJ87, Wei88, SMFK91]

Idea: extension of CFG where non-terminals can span tuples of non-adjacent strings.

Example: $yield(A) = \langle a^n b^n, c^n d^n \rangle$, with $n \geq 1$.

The rewriting rules tell us how to compute the span of the lefthand side non-terminal from the spans of the righthand side non-terminals.

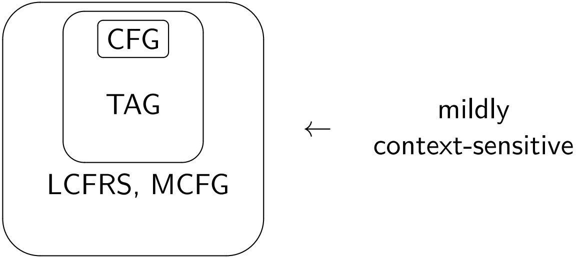
$$A(ab, cd) \rightarrow \varepsilon \quad A(aXb, cYd) \rightarrow A(X, Y) \quad S(XY) \rightarrow A(X, Y)$$

Generated string language: $\{a^n b^n c^n d^n \mid n \geq 1\}$.

LCFRS is more powerful than TAG but still mildly context-sensitive.

Polynomial extensions of CFG (4)

Summary:



In this course, we are interested in mildly context-sensitive formalisms.

Basic Definitions: Languages (1)

Definition 1 (Alphabet, word, language)

1. An alphabet is a nonempty finite set X.
2. A string $x_1 \dots x_n$ with $n \geq 1$ and $x_i \in X$ for $1 \leq i \leq n$ is called a nonempty word on the alphabet X. X^+ is defined as the set of all nonempty words on X.
3. A new element $\varepsilon \notin X^+$ is added: $X^* := x^+ \cup \{\varepsilon\}$.
   For each $w \in X^*$, the concatenation of w and ε is defined as follows: $w\varepsilon = \varepsilon w = w$.
   ε is called the empty word, and each $w \in X^*$ is called a word on X.
4. A set L is called a language iff there is an alphabet X such that $L \subseteq X^*$.

Basic Definitions: Languages (2)

Definition 2 (Homomorphism)

For two alphabets X and Y, a function $f : X^* \to Y^*$ is a homomorphism iff for all $v, w \in X^*$: $f(vw) = f(v)f(w)$.

Definition 3 (Length of a word)

Let X be an alphabet, $w \in X^*$.

1. The length of w, $|w|$ is defined as follows: if $w = \varepsilon$, then $|w| = 0$. If $w = xw'$ for some $x \in X$, then $|w| = 1 + |w'|$.
2. For every $a \in X$, we define $|w|_a$ as the number of as occurring in w: If $w = \varepsilon$, then $|w|_a = 0$, if $w = aw'$ then $|w|_a = |w'|_a + 1$ and if $w = bw'$ for some $b \in X \setminus \{a\}$, then $|w|_a = |w'|_a$.

Basic Definitions: CFG (1)

Definition 4 (Context-free grammar)

A context-free grammar (CFG) is a tuple $G = \langle N, T, P, S \rangle$ such that

1. N and T are disjoint alphabets, the nonterminals and terminals of G.
2. $P \subset N \times (N \cup T)^*$ is a finite set of productions (also called rewriting rules). A production $\langle A, \alpha \rangle$ is usually written $A \rightarrow \alpha$.
3. $S \in N$ is the start symbol.

Basic Definitions: CFG (2)

Definition 5 (Language of a CFG)

Let $G = \langle N, T, P, S \rangle$ be a CFG. The (string) language $L(G)$ of G is the set $\{w \in T^* \mid S \stackrel{*}{\Rightarrow} w\}$ where

- for $w, w' \in (N \cup T)^*$: $w \Rightarrow w'$ iff there is a $A \rightarrow \alpha \in P$ and there are $v, u \in (N \cup T)^*$ such that $w = vAu$ and $w' = v\alpha u$.
- $\stackrel{*}{\Rightarrow}$ is the reflexive transitive closure of $\Rightarrow$:
  - $w \stackrel{0}{\Rightarrow} w$ for all $w \in (N \cup T)^*$, and
  - for all $w, w' \in (N \cup T)^*$: $w \stackrel{n}{\Rightarrow} w'$ iff there is a v such that $w \Rightarrow v$ and $v \stackrel{n-1}{\Rightarrow} w'$.
  - for all $w, w' \in (N \cup T)^*$: $w \stackrel{*}{\Rightarrow} w'$ iff there is a $i \in \mathbb{N}$ such that $w \stackrel{i}{\Rightarrow} w'$.

A language L is called context-free iff there is a CFG G such that $L = L(G)$.

Basic Definitions: CFG (3)

Proposition 1 (Pumping lemma for context-free languages)

Let L be a context-free language. Then there is a constant c such that for all $w \in L$ with $|w| \geq c$: $w = xv_1yv_2z$ with

- $|v_1v_2| \geq 1$,
- $|v_1yv_2| \leq c$, *and*
- *for all* $i \geq 0$: $xv_1^iyv_2^iz \in L$.

Basic Definitions: CFG (4)

Proposition 2

Context-free languages are closed under homomorphisms, i.e., for alphabets T_1, T_2 *and for every context-free language* $L_1 \subset T_1^*$ *and every homomorphism* $h : T_1^* \to T_2^*$, $h(L_1) = \{h(w) \mid w \in L_1\}$ *is a context-free language.*

Proposition 3

Context-free languages are closed under intersection with regular languages, i.e., for every context-free language L *and every regular language* L_r, $L \cap L_r$ *is a context-free language.*

Proposition 4

The copy language $\{ww \mid w \in \{a, b\}^*\}$ *is not context-free.*

Basic Definitions: Trees (1)

Definition 6 (Directed Graph)

1. A directed graph is a pair $\langle V, E\rangle$ where V is a finite set of vertices and $E \subseteq V \times V$ is a set of edges.
2. For every $v \in V$, we define the in-degree of v as $|\{v' \in V \mid \langle v', v\rangle \in E\}|$ and the out-degree of v as $|\{v' \in V \mid \langle v, v'\rangle \in E\}|$.

E^+ is the transitive closure of E and E^* is the reflexive transitive closure of E.

Basic Definitions: Trees (2)

Definition 7 (Tree)

A tree is a triple $\gamma = \langle V, E, r \rangle$ such that

- $\langle V, E \rangle$ is a directed graph and $r \in V$ is a special node, the root node.
- γ contains no cycles, i.e., there is no $v \in V$ such that $\langle v, v \rangle \in E^+$,
- only the root $r \in V$ has in-degree 0,
- every vertex $v \in V$ is accessible from r, i.e., $\langle r, v \rangle \in E^*$, and
- all nodes $v \in V - \{r\}$ have in-degree 1.

A vertex with out-degree 0 is called a leaf. The vertices in a tree are also called nodes.

Basic Definitions: Trees (3)

Definition 8 (Ordered Tree)

A tree is ordered if it has an additional linear precedence relation $\prec \in V \times V$ such that

- $\prec$ is irreflexive, antisymmetric and transitive,
- for all v_1, v_2 with $\{\langle v_1, v_2\rangle, \langle v_2, v_1\rangle\} \cap E^* = \emptyset$: either $v_1 \prec v_2$ or $v_2 \prec v_1$ and if there is either a $\langle v_3, v_1\rangle \in E$ with $v_3 \prec v_2$ or a $\langle v_4, v_2\rangle \in E$ with $v_1 \prec v_4$, then $v_1 \prec v_2$, and
- nothing else is in $\prec$.

We use Gorn addresses for nodes in ordered trees: The root address is ε, and the jth child of a node with address p has address pj.

Basic Definitions: Trees (4)

Definition 9 (Labeling)

A labeling of a graph $\gamma = \langle V, E \rangle$ over a signature $\langle A_1, A_2 \rangle$ is a pair of functions $l : V \rightarrow A_1$ and $g : E \rightarrow A_2$ with A_1, A_2 possibly distinct.

Definition 10 (Syntactic tree)

Let N and T be disjoint alphabets of non-terminal and terminal symbols. A syntactic tree (over N and T) is an ordered finite labeled tree such that $l(v) \in N$ for each vertex v with out-degree at least 1 and $l(v) \in (N \cup T \cup \{\varepsilon\})$ for each leaf v.

Basic Definitions: Trees (5)

Definition 11 (Tree Language of a CFG)

Let $G = \langle N, T, P, S \rangle$ be a CFG.

1. A syntactic tree $\langle V, E, r \rangle$ over N and T is a parse tree in G iff
   - $l(v) \in (T \cup \{\varepsilon\})$ for each leaf v,
   - for every $v_0, v_1, \dots, v_n \in V$, $n \geq 1$ such that $\langle v_0, v_i \rangle \in E$ for $1 \leq i \leq n$ and $\langle v_i, v_{i+1} \rangle \in \prec$ for $1 \leq i < n$, $l(v_0) \to l(v_1) \dots l(v_n) \in P$.
2. A parse tree $\langle V, E, r \rangle$ is a derivation tree in G iff $l(r) = S$.
3. The tree language of G is

$$L_T(G) = \{\gamma \,|\, \gamma \text{ is a derivation tree in } G\}$$

Basic Definitions: Trees (6)

Definition 12 (Weak and Strong Equivalence)

Let F_1, F_2 be two grammar formalisms.

- F_1 and F_2 are weakly equivalent iff for each instance G_1 of F_1 there is an instance G_2 of F_2 that generates the same string language and vice versa.
- F_1 and F_2 are strongly equivalent iff for both formalisms the notion of a tree language is defined and, furthermore, for each instance G_1 of F_1 there is an instance G_2 of F_2 that generates the same tree language and vice versa.

References I

[HU79] John E. Hopcroft and Jeffrey D. Ullman.
Introduction to Automata Theory, Languages and Computation.
Addison Wesley, 1979.

[JLT75] Aravind K. Joshi, Leon S. Levy, and Masako Takahashi.
Tree Adjunct Grammars.
Journal of Computer and System Science, 10:136–163, 1975.

[Jos85] Aravind K. Joshi.
Tree adjoining grammars: How much contextsensitivity is required to provide reasonable structural descriptions?
In D. Dowty, L. Karttunen, and A. Zwicky, editors, *Natural Language Parsing*, pages 206–250. Cambridge University Press, 1985.

[JS97] Aravind K. Joshi and Yves Schabes.
Tree-Adjoning Grammars.
In G. Rozenberg and A. Salomaa, editors, *Handbook of Formal Languages*, pages 69–123. Springer, Berlin, 1997.

[Kal10] Laura Kallmeyer.
Parsing Beyond Context-Free Grammars.
Cognitive Technologies. Springer, Heidelberg, 2010.

[SBMSN87] Walter J. Savitch, Emmon Bach, William Marxh, and Gila Safran-Naveh, editors.
The Formal Complexity of Natural Language.
Studies in Linguistics and Philosophy. Reidel, Dordrecht, Holland, 1987.

[Shi85] Stuart M. Shieber.
Evidence against the context-freeness of natural language.
Linguistics and Philosophy, 8:333–343, 1985.
Reprinted in [SBMSN87].

References II

[SMFK91] Hiroyuki Seki, Takahashi Matsumura, Mamoru Fujii, and Tadao Kasami.
On multiple context-free grammars.
Theoretical Computer Science, 88(2):191–229, 1991.

[VSWJ87] K. Vijay-Shanker, David J. Weir, and Aravind K. Joshi.
Characterizing structural descriptions produced by various grammatical formalisms.
In *Proceedings of ACL*, Stanford, 1987.

[Wei88] David J. Weir.
Characterizing Mildly Context-Sensitive Grammar Formalisms.
PhD thesis, University of Pennsylvania, 1988.