Parsing Beyond CFG
Homework 9: LCFRS 2

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Question 1 *Show that the language* $\{(ag)^n f^m b^n f^m c^n f^m d^n f^m e^n \mid n, m \geq 0\}$ *is not a 2-MCFL.*

You can assume that we have already shown that $\{a^n b^n c^n d^n e^n \mid n \geq 0\}$ *is not a 2-MCFL (see Prop.2 on slide 12).*

Solution:

We assume that $\{(ag)^n f^m b^n f^m c^n f^m d^n f^m e^n \mid n, m \geq 0\}$ is a 2-MCFL. Then, since 2-MCFLs are closed under homomorphisms, the image of this language under h with $h(a) = a, h(b) = b, h(c) = c, h(d) = d$ and $h(e) = h(f) = h(g) = \varepsilon$ must also be a 2-MCFL. But this image is $\{a^n b^n c^n d^n e^n \mid n \geq 0\}$. Contradiction, consequently our assumption does not hold.

Question 2 *Show that for every* $k > 0$ *is holds that the language* $\{w^{2k+1} \mid w \in \{a, b\}^*\}$ *is not a* k*-MCFL (=* k*-LCFRL).*

Solution:

Let $k > 0$ be fixed.

We assume that the language $L = \{w^{2k+1} \mid w \in \{a, b\}^*\}$ is a k-MCFL.

Then its intersection with the regular language $(a^+ b^+)^{2k+1}$ must be a k-MCFL as well. This intersection yields $L' = \{(a^n b^m)^{2k+1} \mid n, m > 0\}$.

If L' is a k-MCFL, then it satisfies the pumping lemma for k-MCFLs. I.e., there must be a word $(a^n b^m)^{2k+1}$ for some $m, n > 0$ such that $2k$ substrings of this word can be iterated. At least one of these strings is not empty and none of them can contain different terminals, otherwise pumping would immediately lead to strings not in L'. However, if at most $2k$ strings are pumped, each of them containing only as or only bs, we necessarily obtain strings that are not in the language L'. This contradicts the assumption that L' satisfies the pumping lemma for k-MCFLs.

Consequently, neither L' nor L are k-MCFLs.