

Parsing Beyond Context-Free Grammars: LCFRS Parsing

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Overview

1. Ranges
2. CYK Parsing
3. Incremental Earley Parsing

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Ranges (1)

- During parsing we have to link the terminals and variables in our LCFRS rules to portions of the input string.
- These can be characterized by their start and end positions.
- A *range* is an pair of indices $\langle i, j \rangle$ that characterizes the span of a component within the input and a range vector characterizes a tuple in the yield of a non-terminal.
- The range instantiation of a rule specifies the computation of an element from the lefthand side yield from elements of in the yields of the right-hand side non-terminals based on the corresponding range vectors.

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Ranges (2)

Example: Rule $A(aXa, bYb) \rightarrow B(X)C(Y)$ and input string $abababcb$.

We assume without loss of generality that our LCFRSs are monotone and ε -free. Furthermore, because of the linearity, the components of a tuple in the yield of an LCFRS non-terminal are necessarily non-overlapping. Then, given this input, we have the following possible instantiations for this rule:

$$\begin{aligned}
 A({}_0aba_3, {}_3bab_6) &\rightarrow B({}_1b_2, {}_4a_5) & A({}_0aba_3, {}_3babcb_8) &\rightarrow B({}_1b_2, {}_4abc_7) \\
 A({}_0aba_3, {}_5bcb_8) &\rightarrow B({}_1b_2, {}_6c_7) & A({}_0ababa_5, {}_5bcb_8) &\rightarrow B({}_1bab_4, {}_6c_7) \\
 A({}_2aba_5, {}_5bcb_8) &\rightarrow B({}_3b_4, {}_6c_7)
 \end{aligned}$$

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Ranges (3)

Definition 1 (Range instantiation, [Boullier, 2000]) Let $G = (N, T, V, P, S)$ be a LCFRS, $w = t_1 \dots t_n \in T^n$ ($n \geq 0$) and $r = A(\vec{\alpha}) \rightarrow A_1(\vec{\alpha}_1) \dots A_m(\vec{\alpha}_m) \in P$ ($0 \leq m$). A *range instantiation* of r wrt. w is a function $f : V \cup \{Eps_i \mid \vec{\alpha}(i) = \varepsilon\} \cup \{t' \mid t' \text{ an occurrence of some } t \in T \text{ in } \vec{\alpha}\} \rightarrow \{(i, j) \mid 0 \leq i \leq j \leq n\}$ such that

- for all occurrences t' of a $t \in T$ in $\vec{\alpha}$, $f(t') = \langle i - 1, i \rangle$ for some i with $t_i = t$,
- for all x, y adjacent in one of the $\vec{\alpha}(i)$ there are i, j, k with $f(x) = \langle i, j \rangle, f(y) = \langle j, k \rangle$; we define then $f(xy) = \langle i, k \rangle$,
- for all $Eps \in \{Eps_i \mid \vec{\alpha}(i) = \varepsilon\}$, $f(Eps) = \langle j, j \rangle$ for some j ; we define then for every ε -argument $\vec{\alpha}(i)$ that $f(\vec{\alpha}(i)) = f(Eps_i)$.

$A(f(\vec{\alpha})) \rightarrow A_1(f(\vec{\alpha}_1)) \dots A_m(f(\vec{\alpha}_m))$ with $f(\langle x_1, \dots, x_k \rangle) = \langle f(x_1), \dots, f(x_k) \rangle$ is then called an *instantiated rule*.

CYK Parsing (1)

First introduced in [Seki et al., 1991]; deduction-based definition in, e.g., [Kallmeyer and Maier, 2010].

Idea: Once all elements in the RHS of a an instantiated rule have been found, complete the LHS.

- We start with the terminal symbols: whenever we can find a range instantiation of a rule with rhs ε , we conclude that this rule can be applied (*scan*).
- We parse bottom-up: whenever, for an instantiated rule, all elements in the rhs have been found, we conclude that this rule can be applied and the lhs of the instantiated rule is deduced (*complete*).
- Our input w is in the language iff S with range vector $\langle \langle 0, n \rangle \rangle$ is in the final set of results that we have deduced.

CYK Parsing (2)

Deduction rules:

Items $[A, \vec{\rho}]$ with $A \in N$, $\vec{\rho}$ is a $\dim(A)$ -dimensional range vector in w .

Axioms (scan): $\frac{}{[A, \vec{\rho}]} A(\vec{\rho}) \rightarrow \varepsilon$ a range instantiated rule

Complete: $\frac{[A_1, \vec{\rho}_1], \dots, [A_m, \vec{\rho}_m]}{[A, \vec{\rho}]} A(\vec{\rho}) \rightarrow A_1(\vec{\rho}_1), \dots, A_m(\vec{\rho}_m)$ a range instantiated rule

Goal item: $[S, \langle \langle 0, n \rangle \rangle]$

CYK Parsing (3)

Deduction rules for binarized ε -free grammars where, without loss of generality, either the lhs contains a single terminal and the rhs is ε or the rule contains only variables:

Items and goal as before.

Scan: $\frac{}{[A, \langle \langle i, i + 1 \rangle \rangle]} A(w_{i+1}) \rightarrow \varepsilon \in P$

Unary: $\frac{[B, \vec{\rho}]}{[A, \vec{\rho}]} A(\vec{\alpha}) \rightarrow B(\vec{\alpha}) \in P$

Binary: $\frac{[B, \vec{\rho}_B], [C, \vec{\rho}_C]}{[A, \vec{\rho}_A]} A(\vec{\rho}_A) \rightarrow B(\vec{\rho}_B)C(\vec{\rho}_C)$ is a range instantiated rule

CYK Parsing (4)

Complexity of CYK parsing with binarized LCFRSs:

We have to consider the maximal number of possible applications of the complete rule.

Binary: $\frac{[B, \vec{\rho}_B], [C, \vec{\rho}_C]}{[A, \vec{\rho}_A]} \quad A(\vec{\rho}_A) \rightarrow B(\vec{\rho}_B)C(\vec{\rho}_C)$
is a range instantiated rule

If k is the maximal fan-out in the LCFRS, we have maximal $2k$ range boundaries in each of the antecedent items of this rule. For variables X_1, X_2 being in the same lhs side argument of the rule, X_1 left of X_2 and no other variables in between, the right boundary of X_1 is the left boundary of X_2 . In the worst case, A, B, C all have fan-out k and each lhs argument contains two variables. This gives $3k$ independent range boundaries and consequently a time complexity of $\mathcal{O}(n^{3k})$ for the entire algorithm.

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Incremental Earley Parsing

Strategy:

- Process LHS arguments incrementally, starting from an *S*-rule
- Whenever we reach a variable, move into rule of corresponding rhs non-terminal (**predict** or **resume**).
- Whenever we reach the end of an argument, **suspend** the rule and move into calling parent rule.
- Whenever we reach the end of the last argument **convert** item into a passive one and **complete** parent item.

This parser is described in [Kallmeyer and Maier, 2009] and inspired by the Thread Automata in [Villemonte de La Clergerie, 2002]

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Incremental Earley Parsing: Items

Passive items: $[A, \vec{\rho}]$ where A is a non-terminal of fan-out k and $\vec{\rho}$ is a range vector of fan-out k

Active items:

$$[A(\vec{\phi}) \rightarrow A_1(\vec{\phi}_1) \dots A_m(\vec{\phi}_m), pos, \langle i, j \rangle, \vec{\rho}]$$

where

- $A(\vec{\phi}) \rightarrow A_1(\vec{\phi}_1) \dots A_m(\vec{\phi}_m) \in P$;
- $pos \in \{0, \dots, n\}$: We have reached input position pos ;
- $\langle i, j \rangle \in \mathbb{N}^2$: We have reached the j th element of i th argument (dot position);
- $\vec{\rho}$ is a range vector containing variable and terminal bindings. All elements are initialized to “?”, an initialized vector is called $\vec{\rho}_{init}$.

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Incremental Earley Parsing: Example (1)

$$S(X_1 X_2) \rightarrow A(X_1, X_2) \quad A(a X_1, b X_2) \rightarrow A(X_1, X_2) \quad A(a, b) \rightarrow \varepsilon$$

Parsing trace for input $w = aabb$:

	pos	item	$\vec{\rho}$	
1	0	$S(\bullet X_1 X_2) \rightarrow A(X_1, X_2)$	$(?, ?)$	axiom
2	0	$A(\bullet a X_1, b X_2) \rightarrow A(X_1, X_2)$	$(?, ?, ?, ?)$	predict, 1
3	0	$A(\bullet a, b) \rightarrow \varepsilon$	$(?, ?)$	predict, 1
4	1	$A(a \bullet X_1, b X_2) \rightarrow A(X_1, X_2)$	$((0, 1), ?, ?, ?)$	scan, 2
5	1	$A(a \bullet, b) \rightarrow \varepsilon$	$((0, 1), ?)$	scan, 3
6	1	$A(\bullet a X_1, b X_2) \rightarrow A(X_1, X_2)$	$(?, ?, ?, ?)$	predict, 4
7	1	$A(\bullet a, b) \rightarrow \varepsilon$	$(?, ?)$	predict 4
8	1	$S(X_1 \bullet X_2) \rightarrow A(X_1, X_2)$	$((0, 1), ?)$	susp. 5, 1
9	1	$A(a, \bullet b) \rightarrow \varepsilon$	$((0, 1), ?)$	resume 5, 8

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Incremental Earley Parsing: Example (2)

10	2	$A(a \bullet X_1, bX_2) \rightarrow A(X_1, X_2)$	$((1, 2), ?, ?, ?)$	scan 6
11	2	$A(a\bullet, b) \rightarrow \varepsilon$	$((1, 2), ?)$	scan 7
12	2	$A(\bullet aX_1, bX_2) \rightarrow A(X_1, X_2)$	$(?, ?, ?, ?)$	predict 10
13	2	$A(\bullet a, b) \rightarrow \varepsilon$	$(?, ?)$	predict 10
14	2	$A(aX_1\bullet, bX_2) \rightarrow A(X_1, X_2)$	$((0, 1), \langle 1, 2 \rangle, ?, ?)$	susp. 11, 4
15	2	$S(X_1 \bullet X_2) \rightarrow A(X_1, X_2)$	$((0, 2), ?)$	susp. 14, 1
16	2	$A(aX_1, \bullet bX_2) \rightarrow A(X_1, X_2)$	$((0, 1), \langle 1, 2 \rangle, ?, ?)$	resume 14, 15
17	3	$A(aX_1, b \bullet X_2) \rightarrow A(X_1, X_2)$	$((0, 1), \langle 1, 2 \rangle, \langle 2, 3 \rangle, ?)$	scan 16
18	3	$A(a, \bullet b) \rightarrow \varepsilon$	$((1, 2), ?)$	resume 11, 17

Incremental Earley Parsing: Example (3)

19	4	$A(a, b\bullet) \rightarrow \varepsilon$	$((1, 2), \langle 3, 4 \rangle)$	scan 18
20	4	$A(\langle 1, 2 \rangle, \langle 3, 4 \rangle)$		convert 19
21	4	$A(aX_1, bX_2\bullet) \rightarrow A(X_1, X_2)$	$((0, 1), \langle 1, 2 \rangle, \langle 2, 3 \rangle, \langle 3, 4 \rangle)$	compl. 17, 20
22	4	$A(\langle 0, 2 \rangle, \langle 2, 4 \rangle)$		convert 21
23	4	$S(X_1X_2\bullet) \rightarrow A(X_1, X_2)$	$((0, 2), \langle 2, 4 \rangle)$	compl. 15, 22
24	4	$S(\langle 0, 4 \rangle)$		convert 23

Incremental Earley Parsing: Deduction Rules

- Notation:
 - $\vec{\rho}(X)$: range bound to variable X .
 - $\vec{\rho}(\langle i, j \rangle)$: range bound to j th element of i th argument on LHS.
- Applying a range vector $\vec{\rho}$ containing variable bindings for given rule c to the argument vector of the lefthand side of c means mapping the i th element in the arguments to $\vec{\rho}(i)$ and concatenating adjacent ranges. The result is defined iff every argument is thereby mapped to a range.

Incremental Earley Parsing: Initialize, Goal item

Initialize: $\frac{}{[S(\vec{\phi}) \rightarrow \vec{\Phi}, 0, \langle 1, 0 \rangle, \vec{\rho}_{init}]} S(\vec{\phi}) \rightarrow \vec{\Phi} \in P$

Goal Item: $[S(\vec{\phi}) \rightarrow \vec{\Phi}, n, \langle 1, j \rangle, \psi]$ with $|\vec{\phi}(1)| = j$ (i.e., dot at the end of lhs argument).

Incremental Earley Parsing: Scan

If next symbol after dot is next terminal in input, scan it.

$$\text{Scan: } \frac{[A(\vec{\phi}) \rightarrow \vec{\Phi}, pos, \langle i, j \rangle, \vec{\rho}]}{[A(\vec{\phi}) \rightarrow \vec{\Phi}, pos + 1, \langle i, j + 1 \rangle, \vec{\rho}']} \vec{\phi}(i, j + 1) = w_{pos+1}$$

where $\vec{\rho}'$ is $\vec{\rho}$ updated with $\vec{\rho}'(i, j + 1) = \langle pos, pos + 1 \rangle$.

Incremental Earley Parsing: Predict

Whenever our dot is left of a variable that is the first argument of some rhs non-terminal B , we predict new B -rules:

$$\text{Predict: } \frac{[A(\vec{\phi}) \rightarrow \dots B(X, \dots) \dots, pos, \langle i, j \rangle, \vec{\rho}_A]}{[B(\vec{\psi}) \rightarrow \vec{\Psi}, pos, \langle 1, 0 \rangle, \vec{\rho}_{init}]}$$

where $\vec{\phi}(i, j + 1) = X, B(\vec{\psi}) \rightarrow \vec{\Psi} \in P$

Incremental Earley Parsing: Suspend

Suspend:

$$\frac{[B(\vec{\psi}) \rightarrow \vec{\Psi}, pos', \langle i, j \rangle, \vec{\rho}_B], [A(\vec{\phi}) \rightarrow \dots B(\vec{\xi}) \dots, pos, \langle k, l \rangle, \vec{\rho}_A]}{[A(\vec{\phi}) \rightarrow \dots B(\vec{\xi}) \dots, pos', \langle k, l + 1 \rangle, \vec{\rho}']}$$

where

- the dot in the antecedent A -item precedes the variable $\vec{\xi}(i)$,
- $|\vec{\psi}(i)| = j$ (i th argument has length j , i.e., is completely processed),
- $|\vec{\psi}| < i$ (i th argument is not the last argument of B),
- $\vec{\rho}_B(\vec{\psi}(i)) = \langle pos, pos' \rangle$
- and for all $1 \leq m < i$: $\vec{\rho}_B(\vec{\psi}(m)) = \vec{\rho}_A(\vec{\xi}(m))$.

$\vec{\rho}$ is $\vec{\rho}_A$ updated with $\vec{\rho}_A(\vec{\xi}(i)) = \langle pos, pos' \rangle$.

Incremental Earley Parsing: Convert

Whenever we arrive at the end of the last argument, we convert the item into a passive one:

$$\text{Convert: } \frac{[B(\vec{\psi}) \rightarrow \vec{\Psi}, pos, \langle i, j \rangle, \vec{\rho}_B]}{[B, \rho]} \quad |\vec{\psi}(i)| = j, |\vec{\psi}| = i, \vec{\rho}_B(\vec{\psi}) = \rho$$

Incremental Earley Parsing: Complete

Whenever we have a passive B item we can use it to move the dot over the variable of the last argument of B in a parent A -rule:

Complete:
$$\frac{[B, \vec{\rho}_B], [A(\vec{\phi}) \rightarrow \dots B(\vec{\xi}) \dots, pos, \langle k, l \rangle, \vec{\rho}_A]}{[A(\vec{\phi}) \rightarrow \dots B(\vec{\xi}) \dots, pos', \langle k, l + 1 \rangle, \vec{\rho}]}$$
 where

- the dot in the antecedent A -item precedes the variable $\vec{\xi}(|\vec{\rho}_B|)$,
- the last range in $\vec{\rho}_B$ is $\langle pos, pos' \rangle$,
- and for all $1 \leq m < |\vec{\rho}_B|$: $\vec{\rho}_B(m) = \vec{\rho}_A(\vec{\xi}(m))$.

$\vec{\rho}$ is $\vec{\rho}_A$ updated with $\vec{\rho}_A(\vec{\xi}(|\vec{\rho}_B|)) = \langle pos, pos' \rangle$.

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Incremental Earley Parsing: Resume

Whenever we are left of a variable that is not the first argument of one of the rhs non-terminals, we resume the rule of the rhs non-terminal.

Resume:
$$\frac{[A(\vec{\phi}) \rightarrow \dots B(\vec{\xi}) \dots, pos, \langle i, j \rangle, \vec{\rho}_A], [B(\vec{\psi}) \rightarrow \vec{\Psi}, pos', \langle k - 1, l \rangle, \vec{\rho}_B]}{[B(\vec{\psi}) \rightarrow \vec{\Psi}, pos, \langle k, 0 \rangle, \vec{\rho}_B]}$$

where

- $\vec{\phi}(i, j + 1) = \vec{\xi}(k)$, $k > 1$ (the next element is a variable that is the k th element in $\vec{\xi}$, i.e., the k th argument of B),
- $|\vec{\psi}(k - 1)| = l$, and
- $\vec{\rho}_A(\vec{\xi}(m)) = \vec{\rho}_B(\vec{\psi}(m))$ for all $1 \leq m \leq k - 1$.

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