Machine Learning
Exercises: MaxEnt classifier

Laura Kallmeyer

Summer 2016, Heinrich-Heine-Universität Düsseldorf

Exercise 1 *Consider the following MaxEnt classifier for documents $d \in \{a, b\}^*$.*

Features:
$$f_1(c, x) = \begin{cases} 1 \text{ if } |x|_a > 0, c = A \\ 0 \text{ otherwise} \end{cases}$$

$$f_2(c, x) = \begin{cases} 1 \text{ if } |x|_b > 0, c = B \\ 0 \text{ otherwise} \end{cases}$$

Weights: $w_1 = 1.4, w_2 = 0.7$.

1. *Consider the following documents: $d_1 = aaaa$, $d_2 = bbbba$, $d_3 = bb$.*

   What are the classes assigned by our classifier to these three?

2. *What are the probabilities of d_1, d_2 and d_3 being in class A?*

Solution:

1. In order to assign the class, it is sufficient to compare the weighted feature sums:

   $$d_1: \sum_{i=1}^{2} w_i f_i(A, d_1) = 1.4, \quad \sum_{i=1}^{2} w_i f_i(B, d_1) = 0$$

   $$d_2: \sum_{i=1}^{2} w_i f_i(A, d_1) = 1.4, \quad \sum_{i=1}^{2} w_i f_i(B, d_1) = 0.7$$

   $$d_3: \sum_{i=1}^{2} w_i f_i(A, d_1) = 0, \quad \sum_{i=1}^{2} w_i f_i(B, d_1) = 0.7$$

   Consequently, the classifier assigns A to d_1 and to d_2 and B to d_3.

2. $P(A|d_1) = \frac{e^{w_1 f_1(A, d_1) + w_2 f_2(A, d_1)}}{e^{w_1 f_1(A, d_1) + w_2 f_2(A, d_1)} + e^{w_1 f_1(B, d_1) + w_2 f_2(B, d_1)}} = \frac{e^{1.4}}{e^{1.4} + 1} = 0.8$

   $P(A|d_2) = \frac{e^{1.4}}{e^{1.4} + e^{0.7}} = 0.67$

   $P(A|d_3) = \frac{1}{1 + e^{0.7}} = 0.33$

Exercise 2 *Assume that we want to build a MaxEnt classifier that assigns a class $c \in \{X, Y\}$ to symbols $x \in \{a, b, c\}$ that are part of a sequence $w \in \{a, b, c\}^+$.*

Our training data:

observed sequence	a	a	b	a	b	a	c	c	a	b	c	a	b	a
classes	X	X	X	Y	X	Y	Y	X	X	X	Y	X	X	Y

What would be reasonable features for our classifier? Only indicator functions are allowed, i.e., features with values 0 or 1.

Solution: The neighborhood seems to be important where both the preceding and the following symbol play a role for the class of an element.

Possible indicator functions:

$$f_{t,c}(y, x) = \begin{cases} 1 \text{ if } t = x \text{ and } y = c \\ 0 \text{ otherwise} \end{cases}$$

$$f_{tl,c}(y, x) = \begin{cases} 1 \text{ if } t \text{ precedes } x \text{ and } y = c \\ 0 \text{ otherwise} \end{cases}$$

$$f_{tr,c}(y, x) = \begin{cases} 1 \text{ if } x \text{ precedes } t \text{ and } y = c \\ 0 \text{ otherwise} \end{cases}$$

for all $t \in \{a, b, c\}$ and $c \in \{X, Y\}$.