
Mildly Context-Sensitive Grammar Formalisms: LCFRS: Relations to other Formalisms

Laura Kallmeyer
Sommersemester 2011

Overview

1. CFG and LCFRS
2. TAG and LCFRS
3. Set-local MCTAG and LCFRS
4. Minimalist Grammar and LCFRS
 - (a) MG
 - (b) MG and LCFRS
5. Other formalisms equivalent to LCFRS

CFG and LCFRS (1)

Every CFG is a simple 1-RCG and vice versa [Boullier, 2000], only with a slightly different syntax:

Construction of a 1-LCFRS for a given CFG: write every CFG production $A \rightarrow X_1 \dots X_k$ as a LCFRS rule $\bar{A}(X_1 \dots X_k) \rightarrow \gamma$ where γ is the concatenation of all $\bar{X}_i(X_i)$ where $1 \leq i \leq k$ and $X_i \in N$. The start predicate is \bar{S} .

	1-LCFRS:
CFG:	
Example: $S \rightarrow aSb$	$\bar{S}(aSb) \rightarrow \bar{S}(S)$
$S \rightarrow \varepsilon$	$\bar{S}(\varepsilon) \rightarrow \varepsilon$

CFG and LCFRS (2)

Construction of a CFG for a given 1-LCFRS: write every rule $A(\alpha) \rightarrow A_1(X_1) \dots A_k(X_k)$ as a CFG rule $A \rightarrow f(\alpha)$ where f is a homomorphism with $f(a) = a$ for all $a \in T$ and $f(X_i) = A_i$ for all $1 \leq i \leq k$.

Example:

1-LCFRS:		CFG:
$S(aXbY) \rightarrow S(X)A(Y)$	$S(\varepsilon) \rightarrow \varepsilon$	$S \rightarrow aSbA \quad S \rightarrow \varepsilon$
$A(cX) \rightarrow A(X)$	$A(\varepsilon) \rightarrow \varepsilon$	$A \rightarrow cA \quad A \rightarrow \varepsilon$

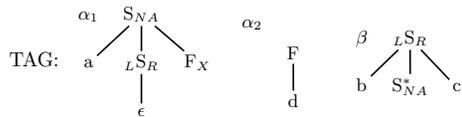
Proposition 1 *For a language L there is a CFG G with $L = L(G)$ iff there is a 1-LCFRS G' with $L = L(G')$.*

TAG and LCFRS (1)

General idea of the transformation of a TAG into an equivalent LCFRS [Boullier, 1998]:

- The LCFRS contains non-terminals $\langle \alpha \rangle(X)$ and $\langle \beta \rangle(L, R)$ for initial trees α and auxiliary trees β respectively.
- X covers the yield of α and all trees added to α , while L and R cover those parts of the yield of β (including all trees added to β) that are to the left and the right of the foot node of β .
- The rules reduce the components of these non-terminals by identifying those parts that come from the elementary tree α/β itself and those parts that come from one of the elementary trees added by substitution or adjunction.

TAG and LCFRS (2)



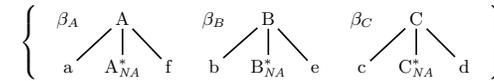
Equivalent LCFRS:

$$\begin{aligned}
 S(X) &\rightarrow \langle \alpha_1 \rangle(X) \mid \langle \alpha_2 \rangle(X) & \langle \alpha_1 \rangle(aX) &\rightarrow \langle \alpha_2 \rangle(X) \\
 \langle \alpha_1 \rangle(aLRX) &\rightarrow \langle \beta \rangle(L, R) \langle \alpha_2 \rangle(X) & \langle \beta \rangle(Lb, cR) &\rightarrow \langle \beta \rangle(L, R) \\
 \langle \alpha_2 \rangle(d) &\rightarrow \epsilon & \langle \beta \rangle(b, c) &\rightarrow \epsilon
 \end{aligned}$$

Proposition 2 For every TAG L there is a 2-LCFRS G with $L = L(G)$.

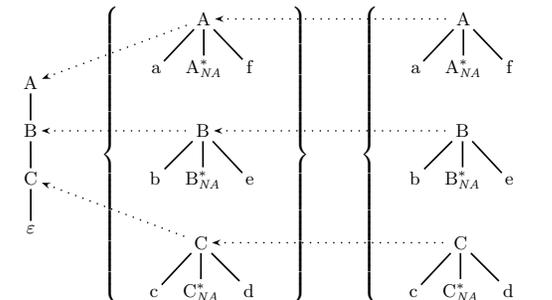
MCTAG and LCFRS (1)

MCTAG example (reminder): α



MCTAG and LCFRS (2)

Derivation for $aabbccddeeff$:



MCTAG and LCFRS (3)

Proposition 3 *Set-local MCTAG and LCFRS are weakly equivalent [Weir, 1988].*

The constructions given below are not exactly the ones from [Weir, 1988].

MCTAG and LCFRS (4)

Construction of an equivalent LCFRS for a given MCTAG:

- We introduce non-terminals for all multicomponent sets. Their fan-out depends on the number of trees and whether they are initial or auxiliary: every initial tree contributes one component while every auxiliary tree contributes two components.
- For every set Γ and all sets $\Gamma_1, \dots, \Gamma_k$ that can attach to Γ such that all obligatory adjunctions and substitutions are performed, we introduce a rule that tells us how the yield of Γ can be obtained from the yields of $\Gamma_1, \dots, \Gamma_k$ and from the terminals occurring in Γ .
- For every set Γ without substitution nodes or OA constraints, we add a terminating rule that lists only the terminals occurring in Γ .

MCTAG and LCFRS (5)

Example: LCFRS that is equivalent to MCTAG from previous slides:

$N = \{\alpha, \beta_{A,B,C}, S\}$, start symbol S and rules

$$S(X) \rightarrow \alpha(X)$$

$$\alpha(\varepsilon) \rightarrow \varepsilon$$

$$\alpha(X_1 X_2 X_3 X_4 X_5 X_6) \rightarrow \beta_{A,B,C}(X_1, X_2, X_3, X_4, X_5, X_6)$$

$$\beta_{A,B,C}(a, b, c, d, e, f) \rightarrow \varepsilon$$

$$\beta_{A,B,C}(X_1 a, X_2 b, X_3 c, d X_4, e X_5, f X_6) \rightarrow \beta_{A,B,C}(X_1, X_2, X_3, X_4, X_5, X_6)$$

MCTAG and LCFRS (6)

Construction of an equivalent MCTAG for a given LCFRS: First, we make sure no non-terminal occurs twice in a rhs and our LCFRS is monotone. Then the construction is as follows:

- For each rule we introduce a multicomponent set that contains an initial tree for each component of the lhs.
- The root of this initial tree is labelled A_k if A is the lhs symbol and the tree describes the k th component.
- The daughters describe the elements of this component from left to right, they are labelled (from left to right) with the terminals from the lhs and with B_i if the lhs element is the i th argument of the rhs element B .
- The MCTAG has a start symbol, namely S_1 .

Note: This construction yields an MCTAG without adjunction.

MCTAG and LCFRS (7)

Example:

$$\begin{aligned} S(XYZ) \rightarrow A(X, Z)B(Y) & \left\{ \begin{array}{c} S_1 \\ / \quad | \quad \backslash \\ A_1 \quad B_1 \quad A_2 \end{array} \right\} \\ A(aXb, cYd) \rightarrow A(X, Y) & \left\{ \begin{array}{c} A_1 \qquad \qquad A_2 \\ / \quad | \quad \backslash \quad , \quad / \quad | \quad \backslash \\ a \quad A_1 \quad b \quad \quad c \quad A_2 \quad d \end{array} \right\} \\ A(ab, \varepsilon) \rightarrow \varepsilon & \left\{ \begin{array}{c} A_1 \\ / \quad | \quad \backslash \\ a \quad \quad \quad b \end{array} \right\}, \left\{ \begin{array}{c} A_2 \\ | \\ \varepsilon \end{array} \right\} \\ B(e) \rightarrow \varepsilon & \left\{ \begin{array}{c} B_1 \\ | \\ e \end{array} \right\} \end{aligned}$$

Minimalist Grammar (1)

- Minimalist Grammars (MGs) were proposed by [Stabler, 1997] as a formalization of Chomsky's Minimalist Program [Chomsky, 1995].
- Roughly, MGs consist of a set of trees together with two operations, *merge* and *move*, that allow us to transform these trees.
- Michaelis [Michaelis, 2001a, Michaelis, 2001b] has shown that MGs are equivalent to LCFRS.

Minimalist Grammar (2)

- An MG consists of a set *Lex* of finite ordered binary trees $\tau = \langle V, E, r \rangle$, so-called *expressions*.
- In such expressions τ , there is an additional relation of *projection* defined among sisters. For every $v_1 \neq v_2$ such that there exists a v with $\langle v, v_1 \rangle, \langle v, v_2 \rangle \in E$, either v_1 projects over v_2 or vice versa.
- Furthermore, all leaves in τ are labeled with a finite sequence of features.

Minimalist Grammar (3)

- A node $v \in V$ in an expression $\tau = \langle V, E, r \rangle$ is called a *maximal projection* if either $v = r$ or its sister projects over v .
- The *head* of a node $v \in V$ is the leaf $h(v)$ such that $\{v' \mid \langle v, v' \rangle \in E^+, \langle v', h(v) \rangle \in E^*\}$ does not contain maximal projections, i.e., the path from v to its head contains only nodes that project over their sisters.

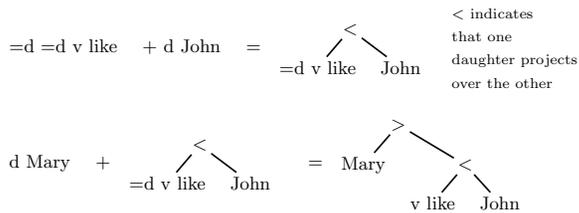
Minimalist Grammar (4)

Besides the set *Lex*, MG provides two operations, *merge* and *move* to create new expressions.

- *Merge* builds a new tree from two existing ones by considering them the two subtrees dominated by a new root node. Its application depends on the head features of the two trees and it modifies these features.
- *Move* transforms a single tree into a new one. Roughly, it consists of extracting a subtree, replacing it with a trace ϵ or deleting its phonetic material in the original place. The extracted subtree and the result of deleting it in the original tree become sisters with a new root node as mother.

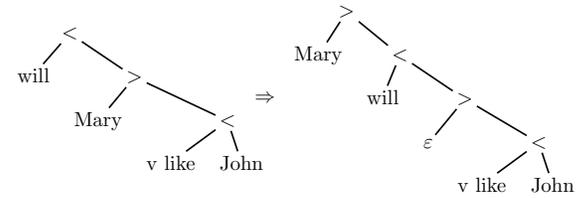
Minimalist Grammar (5)

Examples of *merge*:



Minimalist Grammar (5)

Example of *move*:



MG and LCFRS

From MG to an equivalent LCFRS (Intuition):

- Merge operations amount to concatenation, eventually with a gap in between the two concatenated parts.

$$V'(XY) \rightarrow V(X)DP(Y), \quad VP(X, Y) \rightarrow DP(X)V'(Y),$$

$$T'(X, Y, Z) \rightarrow Aux(X)VP(Y, Z)$$

- Move amounts to a switching of components such that the k th component (for some $k > 1$) becomes the first/is concatenated to the first component.

$$TP(X, Y, Z) \rightarrow T'(Y, X, Z)$$

This leads to non-monotone LCFRS (unordered simple RCG).

Other equivalent formalisms

- Finite-copying Lexical Functional Grammar [Seki et al., 1993]
- Hyperedge Replacement Grammars [Engelfriet and Heyker, 1991]
- Deterministic Tree-Walking Transducers [Weir, 1992]

Grammar Formalisms 21 LCFRS and other formalisms

Kallmeyer Sommersemester 2011

References

- [Boullier, 1998] Boullier, P. (1998). A generalization of mildly context-sensitive formalisms. In *Proceedings of the Fourth International Workshop on Tree Adjoining Grammars and Related Formalisms (TAG+4)*, pages 17–20, University of Pennsylvania, Philadelphia.
- [Boullier, 2000] Boullier, P. (2000). A cubic time extension of context-free grammars. *Grammars*, 3(2/3):111–131.
- [Chomsky, 1995] Chomsky, N. (1995). *The Minimalist Program*. MIT Press.
- [Engelfriet and Heyker, 1991] Engelfriet, J. and Heyker, L. (1991). The string generating power of context-free hypergraph grammars. *Journal of Computer and System Sciences*, 43:328–360.

Grammar Formalisms 22 LCFRS and other formalisms

[Michaelis, 2001a] Michaelis, J. (2001a). Derivational minimalism is mildly context-sensitive. In Moortgat, M., editor, *Logical Aspects of Computational Linguistics*, volume 2014 of *LNCS/LNAI*, pages 179–198, Berlin, Heidelberg. Springer.

[Michaelis, 2001b] Michaelis, J. (2001b). Transforming linear context-free rewriting systems into minimalist grammars. In Philippe de Groote, Glyn Morrill, C. R., editor, *Logical Aspects of Computational Linguistics*, volume 2099 of *LNCS/LNAI*, pages 228–244, Berlin, Heidelberg. Springer.

[Seki et al., 1993] Seki, H., Nakanishi, R., Kaji, Y., Ando, S., and Kasami, T. (1993). Parallel multiple context-free grammars, finite-state translation systems, and polynomial-time recognizable subclasses of lexical-functional grammars. In *31st Meeting of the Association for Computational Linguistics (ACL'93)*, pages 121–129.

Grammar Formalisms 23 LCFRS and other formalisms

Kallmeyer Sommersemester 2011

- [Stabler, 1997] Stabler, E. P. (1997). Derivational Minimalism. In Retoré, C., editor, *Logical Aspects of Computational Linguistics*, number 1328 in *Lecture Notes in Computer Science*, pages 68–95, NY. Springer-Verlag.
- [Weir, 1992] Weir, D. (1992). Linear context-free rewriting systems and deterministic tree-walking transducers. In *Proceedings of the 30th Annual Meeting of the Association for Computational Linguistics*, pages 136–143.
- [Weir, 1988] Weir, D. J. (1988). *Characterizing Mildly Context-Sensitive Grammar Formalisms*. PhD thesis, University of Pennsylvania.

Grammar Formalisms 24 LCFRS and other formalisms