Einführung in die Computerlinguistik

Probabilistic CFG

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Introduction

- Goal: Induce CFGs from training data, for instance from treebanks.
- Extend CFG to probabilistic CFG.
- Compute the likelihood of parse trees and of sentences according to the PCFG.
- Compute the best parse tree for a given sentence (parsing).

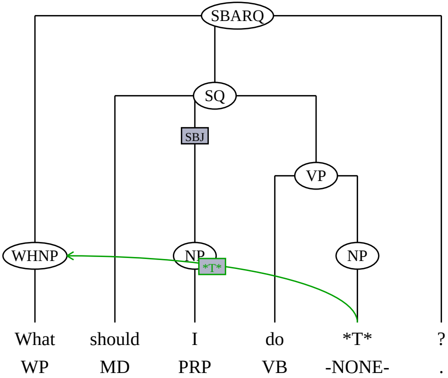
Jurafsky and Martin (2009); Manning and Schütze (1999)
Some of the slides are due to Wolfgang Maier.

Data-Driven Parsing (1)

- Linguistic grammars can not only be created manually. Another way to obtain grammars is to interpret the syntactic structures in a treebank as the derivations of a latent grammar and to use an appopriate algorithm for grammar extraction.
- One can also estimate occurrence probabilities for the rules of a grammar. These can be used to determine the best parse, resp. parses of a sentence.
- Furthermore, rule probabilities can serve to speed up parsing.
- Parsing with a probabilistic grammar obtained from a treebank is called data-driven parsing.

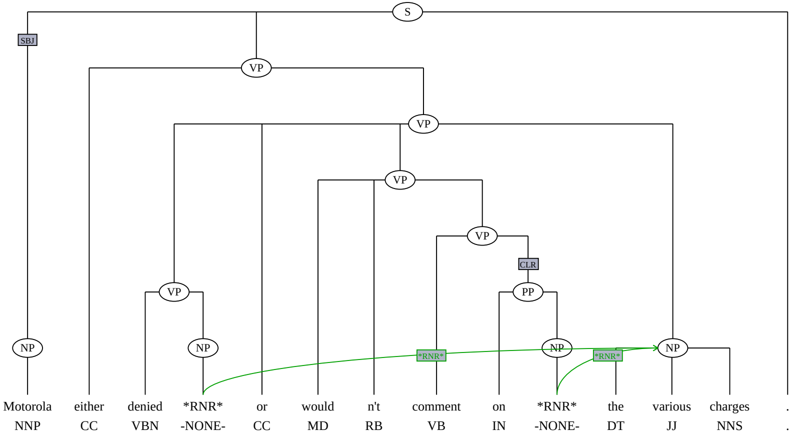
Data-Driven Parsing (2)

Sample tree from the Penn Treebank (PTB):



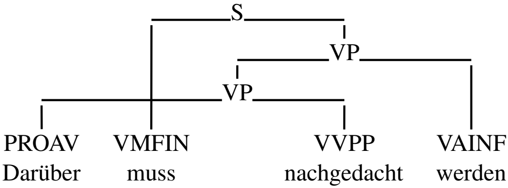
Data-Driven Parsing (3)

Sample tree from the Penn Treebank (PTB):



Data-Driven Parsing (4)

Sample tree from the German treebank Negra:



PCFG (1)

In most cases, probabilistic CFGs are used for data-driven parsing.

A **Probabilistic Context-Free Grammar** (PCFG) is a tuple $G_P = (N, T, P, S, p)$ where (N, T, P, S) is a CFG and $p : P \to [0, 1]$[1] is a function such that for all $A \in N$,

$$\sum_{A \to \alpha \in P} p(A \to \alpha) = 1$$

$p(A \to \alpha)$ is the conditional probability $p(A \to \alpha \mid A)$

[1] $[0, 1]$ denotes $\{i \in \mathbb{R} \mid 0 \leq i \leq 1\}$.

PCFG (2)

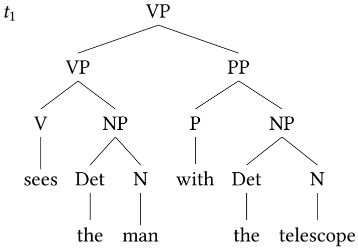
Example:

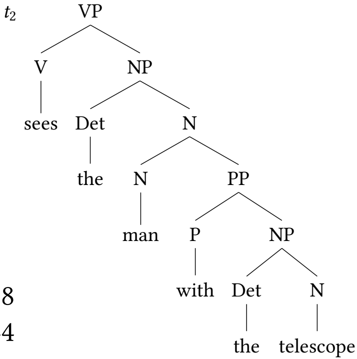
.8	VP $\rightarrow$ V NP	1	V $\rightarrow$ sees
.2	VP $\rightarrow$ VP PP	1	Det $\rightarrow$ the
1	NP $\rightarrow$ Det N	1	P $\rightarrow$ with
1	PP $\rightarrow$ P NP	.6	N $\rightarrow$ man
.1	N $\rightarrow$ N PP	.3	N $\rightarrow$ telescope

Start symbol VP.

PCFG (3)

- Probability of a parse tree: product of the probabilities of the rules used to generate the parse tree.
- Probability of a category A spanning a string w: sum of the probabilities of all parse trees with root label A and yield w.





$$P(t_1) = 0.6 \cdot 0.8 \cdot 0.2 \cdot 0.3 = 0.0288$$
$$P(t_2) = 0.6 \cdot 0.8 \cdot 0.1 \cdot 0.3 = 0.0144$$

$$p(\text{VP,sees the man with the telescope}) = 0.0288 + 0.0144$$

PCFG (4)

Probabilities of leftmost derivations:

Let $G = (N, T, P, S, p)$ be a PCFG, and let $\alpha, \gamma \in (N \cup T)^*$.

- Let $A \to \beta \in P$. The probability of a leftmost derivation $\alpha \overset{A\to\beta}{\Rightarrow_l} \gamma$ is

$$p(\alpha \overset{A\to\beta}{\Rightarrow_l} \gamma) = p(A \to \beta)$$

- Let $A_1 \to \beta_1, \ldots, A_m \to \beta_m \in P$, $m \in \mathbb{N}$. The probability of a leftmost derivation $\alpha \overset{A_1\to\beta_1}{\Rightarrow_l} \cdots \overset{A_m\to\beta_m}{\Rightarrow_l} \gamma$ is

$$p(\alpha \overset{A_1\to\beta_1}{\Rightarrow_l} \cdots \overset{A_m\to\beta_m}{\Rightarrow_l} \gamma) = \prod_{i=1}^{m} p(A_i \to \beta_i)$$

PCFG (5)

- The probability of leftmost deriving γ from α, $\alpha \overset{*}{\Rightarrow}_l \gamma$ is defined as the sum over the probabilities of all leftmost derivations of γ from α:

$$p(\alpha \overset{*}{\Rightarrow}_l \gamma) = \sum_{i=1}^{k} \prod_{j=1}^{m} p(A_j^i \rightarrow \beta_j^i)$$

  where $k \in \mathbb{N}$ is the number of leftmost derivations of γ from α and $m \in \mathbb{N}$ is the derivation length of the ith derivation and $A_j^i \rightarrow \beta_j^i$ is the jth derivation step of the ith leftmost derivation.

In the following, the subscript l is omitted assuming that derivations are identified with the corresponding leftmost derivation for probabilities.

PCFG (6)

A PCFG is consistent if the sum of the probabilities of all sentences in the language equals 1.

Example of an inconsistent PCFG G:

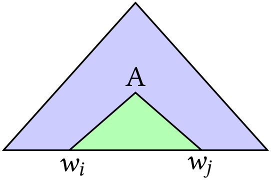
$.4\ S \rightarrow A \quad .6\ S \rightarrow B \quad 1\ A \rightarrow a \quad 1\ B \rightarrow B$

Problem: probability mass disappears into infinite derivations.

$\sum_{w \in L(G)} p(w) = p(a) = 0.4$

Inside and Outside Probability (1)

Idea: given a word $w = w_1 \cdots w_n$ and a category A, we consider the case that A is part of a derivation tree for w such that A spans $w_i \cdots w_j$.



- Inside probability of $\langle A, w_i \cdots w_j \rangle$: probability of a tree with root A and leaves $w_i \cdots w_j$.
- Outside probability of $\langle A, w_i \cdots w_j \rangle$: probability of a tree with root S and leaves $w_1 \cdots w_{i-1} A w_{j+1} \cdots w_n$.

Inside and Outside Probability (2)

Let G be a PCFG and let $w = w_1 \cdots w_n$, $n \in \mathbb{N}$, $w_i \in \Sigma$ for some alphabet Σ, $1 \leq i \leq n$, be an input string. Let $1 \leq i \leq j \leq n$ and $A \in N$.

1. The probability of deriving $w_i \cdots w_j$ from A is called inside probability and defined as

$$p(A \overset{*}{\Rightarrow} w_i \cdots w_j)$$

2. The probability of a deriving A, preceded by $w_1 \cdots w_{i-1}$ and followed by $w_{j+1} \cdots w_n$ in a parse tree rooted with S is called outside probability and defined as

$$p(S \overset{*}{\Rightarrow} w_1 \cdots w_{i-1} A w_{j+1} \cdots w_n)$$

The product of inside and outside probability gives the probability of a parse tree for w containing a non-terminal A that spans $w_i \cdots w_j$.

Inside and Outside Probability (3)

Inside algorithm for computing the inside probabilities of a PCFG $G = (N, T, P, S, p)$ given an input string w:

- We assume all non-terminals $A \in N$ to be continuously numbered from 1 to $|N|$.
- We use a three-dimensional matrix chart α, where the first dimension contains an index denoting a non-terminal, and the second and third dimension contain indices denoting the start and the end of a part of the input string.
- Each cell $[A, i, j]$ in α, written as $\alpha_A(i, j)$ contains the sum of probabilities of all derivations $A \stackrel{*}{\Rightarrow}_l w_i \cdots w_j$.
- We assume our grammar to be in Chomsky Normal Form. I.e., all productions have either the form $A \rightarrow a$ with $a \in T$ or $A \rightarrow BC$ with $B, C \in N$.

Inside and outside probability (4)

Idea of the inside computation: We fill a $|N| \times |w| \times |w|$ matrix α where the first dimension is the id of a non-terminal, and the second and third are the start and end indices of a span. $\alpha_{A,i,j}$ gives the probability of deriving $w_i \dots w_j$ from A or, put differently, of a parse tree with root label A and yield $w_i \dots w_j$:

$$\alpha_{A,i,j} = P(A \stackrel{*}{\Rightarrow} w_i \dots w_j | A)$$

Inside computation

1. for all $1 \leq i \leq |w|$ and $A \in N$:
   if $A \to w_i \in P$, then $\alpha_{A,i,i} = p(A \to w_i)$, else $\alpha_{A,i,i} = 0$
2. for all $1 \leq i < j \leq |w|$ and $A \in N$:
   $\alpha_{A,i,j} = \sum_{A \to BC \in P} \sum_{k=i}^{j-1} p(A \to BC) \alpha_{B,i,k} \alpha_{C,k+1,j}$

We have in particular $\alpha_{S,1,|w|} = P(w)$.

Inside and outside probability (5)

Inside computation

0.3: S → AS   0.6: S → AX   0.1: S → a   1: X → SA   1: A → a

input $w = a^4$

j				
4	$(3.87 \cdot 10^{-2}$,S), (0.069,X)	$(6.9 \cdot 10^{-2}$,S), (0.03,X)	$(3 \cdot 10^{-2}$,S), (0.1,X)	(1,A), (0.1,S)
3	$(6.9 \cdot 10^{-2}$,S), (0.03,X)	$(3 \cdot 10^{-2}$,S), (0.1,X)	(1,A), (0.1,S)	
2	$(3 \cdot 10^{-2}$,S), (0.1,X)	(1,A), (0.1,S)		
1	(1,A), (0.1,S)			
	1	2	3	4 i

$P(aaaa) = \alpha_{S,1,4} = 0.0387$

Inside and outside probability (6)

Outside algorithm: We fill a $|N| \times |w| \times |w|$ matrix β such that $\beta_{A,i,j}$ gives the probability of deriving $w_1 \dots w_{i-1} A w_{j+1} \dots w_{|w|}$ from S or, put differently, of deriving a tree with root label S and yield $w_1 \dots w_{i-1} A w_{j+1} \dots w_{|w|}$:

$$\beta_{A,i,j} = P(S \stackrel{*}{\Rightarrow} w_1 \dots w_{i-1} A w_{j+1} \dots w_{|w|} | S)$$

We need the inside probabilities in order to compute the outside probabilities.

Outside computation

1. $\beta_{S,1,|w|} = 1$ and $\beta_{A,1,|w|} = 0$ for all $A \neq S$
2. for all l with $n > l \geq 1$ (starting with $n-1$):
   for all $1 \leq i < |w| - l + 1$ and $A \in N$:
   $j = i + l - 1$
   $\beta_{A,i,j} = \sum_{B \to AC \in P} \sum_{k=j+1}^{n} p(B \to AC) \beta_{B,i,k} \alpha_{C,j+1,k}$
   $+ \sum_{B \to CA \in P} \sum_{k=1}^{i-1} p(B \to CA) \beta_{B,k,j} \alpha_{C,k,i-1}$

Inside and outside probability (7)

Outside computation

0.3: S → AS 0.6: S → AX 0.1: S → a 1: X → SA 1: A → a

input $w = a^3$

j				
3	(1,S), (0,A), (0,X)	(0.3,S), (0,A), (0.6,X)	$(9 \cdot 10^{-2}$,S), (0.18,X), $(6 \cdot 10^{-2}$,A)	
2	(0,S), (0,X), (0.03,A)	(0.6,S), (0,X), $(9 \cdot 10^{-3}$,A)		
1	(0,S), (0,X), $(6.9 \cdot 10^{-2}$,A)			
	1	2	3	i

Inside and Outside Probability (8)

Probability of a sentence:

- $p(w_1 \cdots w_n) = \alpha_S(1, n)$
- $p(w_1 \cdots w_n) = \sum_A \beta_A(k, k) p(A \rightarrow w_k)$ for any k, $1 \leq k \leq n$
- $p(w_1 \cdots w_n | A \stackrel{*}{\Rightarrow} w_i \cdots w_j) = \beta_A(i, j) \alpha_A(i, j)$

- Inside probability: calculated bottom-up (CYK-style)
- Outside probability: calculated top-down.
- Sentence probability can be calculated in many ways.

Parsing (1)

- In PCFG parsing, we want to compute the most probable parse tree (= most probable derivation) given an input sentence w.
- This means that we are disambiguating: Among several readings, we search for the best.
- Sometimes, the k best are searched for ($k > 1$).
- During parsing, we must make sure that updates on probabilities (because a better derivation has been found for a non-terminal) do not require updates on other parts of the chart. $\Rightarrow$ the order should be such that an item is used within a derivation only when its final probability is reached.

Parsing (2)

We can extend the symbolic CYK parser to a probabilistic one. Instead of summing over all derivations (as in the computation of the inside probability), we keep the best one.

Assume a three-dimensional chart C (non-terminal, start index, length).

$C_{A,i,l} := 0$ `for all` A, i, l
$C_{A,i,1} := p$ `if` $p : A \rightarrow w_i \in P$ **scan**
`for all` $l \in [1..n]$`:`
  `for all` $i \in [1..n-l+1]$`:`
    `for every` $p : A \rightarrow B\ C$`:`
      `for every` $l_1 \in [1..l-1]$`:`
        $C_{A,i,l} = max\{C_{A,i,l}, p \cdot C_{B,i,l_1} \cdot C_{C,i+l_1,l-l_1}\}$ **complete**

Parsing (3)

We extend this to a parser.

- The parser can also deal with unary productions $A \to B$.
- Every chart field has three components, the probability, the rule that has been used and, if the rule is binary, the length l_1 of the first righthand side element.
- We assume that the grammar does not contain any loops $A \stackrel{+}{\Rightarrow} A$.

Parsing (4)

$C_{A,i,1} = \langle p, A \rightarrow w_i, - \rangle$ `if` $p : A \rightarrow w_i \in P$ **scan**

```
for all l ∈ [1..n] and for all i ∈ [1..n − l]:
  for all p : A → B C and for all l1 ∈ [1..l − 1]:
    for all l1 ∈ [1..l − 1]:
      if C_{B,i,l1} ≠ ∅ and C_{C,i+l1,l−l1} ≠ ∅ then:
        p_new = p · C_{B,i,l1}[1] · C_{C,i+l1,l−l1}[1]
        if C_{A,i,l} == ∅ or C_{A,i,l}[1] < p_new then:
          C_{A,i,l} = ⟨p_new, A → BC, l1⟩        binary complete
  repeat until C does not change any more:
    for every p : A → B:
      if C_{B,i,l} ≠ ∅ then:
        p_new = p · C_{B,i,l}[1]
        if C_{A,i,l} == ∅ or C_{A,i,l}[1] < p_new then:
          C_{A,i,l} = ⟨p_new, A → B, −⟩          unary complete
return build_tree(S, 1, n)
```

Parsing (5)

.1	VP $\to$ VP NP	1	NP $\to$ Det N	.3	V $\to$ eats
.6	VP $\to$ V NP	.3	V $\to$ sees	1	Det $\to$ this
.3	VP $\to$ V	.4	V $\to$ comes	.5	N $\to$ morning
.5	N $\to$ apple				

Start symbol VP, input $w =$ *eats this morning*

l				
3	.0045, VP $\to$ VP NP, 1			
2		.5, NP $\to$ Det N, 1		
1	.09, VP $\to$ V .3, V $\to$ eats	1, Det $\to$ this	.5, N $\to$ morning	
	1	2	3	i

Parsing (6)

.1	VP → VP NP	1	NP → Det N	.3	V → eats
.6	VP → V NP	.3	V → sees	1	Det → this
.3	VP → V	.4	V → comes	.5	N → morning
.5	N → apple				

Start symbol VP, input $w =$ *eats this morning*

l				
3	.09, VP → V NP, 1			
2		.5, NP → Det N, 1		
1	.09, VP → V .3, V → eats	1, Det → this	.5, N → morning	
	1	2	3	i

(The analysis of the VP gets revised since a better parse tree has been found.)

Jurafsky, D. and Martin, J. H., editors (2009). *Speech and Language Processing. An Introduction to Natural Language Processing, Computational Linguistics, and Speech Recognition.* Prentice Hall Series in Articial Intelligence. Pearson Education International. Second Edition.

Manning, C. D. and Schütze, H. (1999). *Foundations of Statistical Natural Language Processing.* The MIT Press, Cambridge, Massachusetts, London, England.