

# Einführung in die Computerlinguistik

## Feature Structures – Merkmalstrukturen

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## Introduction (1)

Non-terminals that are used in CFGs are usually not enough to express linguistic generalisations

### Example: Agreement

Missed generalisation:

$S \rightarrow \text{NP-Sg VP-Sg}$     $S \rightarrow \text{NP-Pl VP-Pl}$

Better:  $S \rightarrow \text{NP VP}$    Condition: NP and VP agree in their number

## Introduction (2)

To express such generalisations, we can factorise the non-terminals:

- A non-terminal is no longer atomic, but it has a structure.
- The content of the non-terminals is described via **attributes** (i.e., **features**) that can have certain **values**.
- Such structures are called **attribute-value structures** or **feature structures**. They are often represented in an **attribute-value matrix (AVM)**.

### Feature structures

$$\begin{bmatrix} \text{cat} & \text{NP} \\ \text{num} & \text{Pl} \end{bmatrix}$$
$$\begin{bmatrix} \text{lex} & \text{ihm} \\ \text{cat} & \text{Pro} \\ \text{case} & \text{dat} \\ \text{num} & \text{Sg} \\ \text{gen} & \text{m} \end{bmatrix}$$
$$\begin{bmatrix} \text{pred} & \text{give} \\ \text{donor} & \text{Adam} \\ \text{theme} & \text{apple} \\ \text{recipient} & \text{Eve} \end{bmatrix}$$

## Introduction (2)

- It is possible to refer to the same attribute value in different places (**structure sharing**)

### Structure sharing

$$\left[ \begin{array}{cc} \text{cat} & \text{S} \end{array} \right] \rightarrow \left[ \begin{array}{cc} \text{cat} & \text{NP} \\ \text{num} & \boxed{1} \end{array} \right] \left[ \begin{array}{cc} \text{cat} & \text{VP} \\ \text{num} & \boxed{1} \end{array} \right]$$

(The variable  $\boxed{1}$  always denotes the same value.)

pred	give
donor	$\boxed{1}$ Adam
agent	$\boxed{1}$
theme	apple
recipient	Eve

## Introduction (4)

- **Underspecification:** Not all the values are always known. Instead of listing all the possibilities it is possible to specify only those values that are known.

### Underspecification of attributes

$\begin{bmatrix} \text{cat} & \text{N} \\ \text{num} & \text{Sg} \\ \text{gen} & \text{m} \end{bmatrix} \rightarrow \text{man}$        $\begin{bmatrix} \text{cat} & \text{N} \\ \text{gen} & \text{n} \end{bmatrix} \rightarrow \text{fish}$

$\begin{bmatrix} \text{cat} & \text{Det} \\ \text{num} & \text{Sg} \end{bmatrix} \rightarrow \text{a}$        $\begin{bmatrix} \text{cat} & \text{Det} \end{bmatrix} \rightarrow \text{the}$

$\begin{bmatrix} \text{cat} & \text{NP} \\ \text{num} & \boxed{1} \\ \text{gen} & \boxed{2} \\ \text{pers} & 3 \end{bmatrix} \rightarrow \begin{bmatrix} \text{cat} & \text{Det} \\ \text{num} & \boxed{1} \end{bmatrix} \begin{bmatrix} \text{cat} & \text{N} \\ \text{num} & \boxed{1} \\ \text{gen} & \boxed{2} \end{bmatrix}$

## Introduction (5)

- Attributes do not necessarily have atomic values. The value of an attribute can be another attribute-value structure.

### Recursive feature structures

$$\begin{bmatrix} \text{cat} & \text{N} \\ \text{agr} & \begin{bmatrix} \text{gen} & \text{n} \end{bmatrix} \end{bmatrix} \rightarrow \text{fish} \qquad \begin{bmatrix} \text{cat} & \text{Det} \\ \text{agr} & \begin{bmatrix} \text{num} & \text{Sg} \end{bmatrix} \end{bmatrix} \rightarrow \text{a}$$

$$\begin{bmatrix} \text{cat} & \text{NP} \\ \text{agr} & \boxed{1} \begin{bmatrix} \text{pers} & 3 \end{bmatrix} \end{bmatrix} \rightarrow \begin{bmatrix} \text{cat} & \text{Det} \\ \text{agr} & \boxed{1} \end{bmatrix} \begin{bmatrix} \text{cat} & \text{N} \\ \text{agr} & \boxed{1} \end{bmatrix}$$

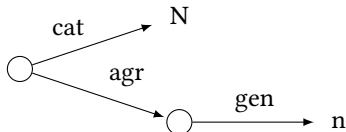
## Attribute-value structures as graphs (1)

Attribute-value structures are usually formalised as directed graphs.

Two possibilities: an attribute-value matrix such as

$$\begin{bmatrix} \text{cat} & \text{N} \\ \text{agr} & \begin{bmatrix} \text{gen} & \text{n} \end{bmatrix} \end{bmatrix}$$

- 1 can be represented as a directed graph



- 2 or as a description of such a graph, that can be in principle satisfied by an infinite number of graphs.

## Attribute-value structures as graphs (2)

In the following, we assume feature structures to be graphs (and not expressions in a feature logic).

### Feature structure

A (untyped) feature structure can be defined as a tuple  $\langle V, A, Val, r \rangle$  such that

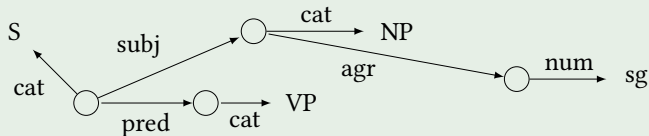
- $V$  is a set of vertices (= nodes).
- $A$  is a finite set of partial functions  $a : V \rightarrow V$
- $Val$  is a finite set of atomic values and there is a partial function  $l_{Val} : \{v \in V \mid \text{there is no } a \in A \text{ such that } a(v) \text{ is defined, i.e., there is no outgoing edge for } v\} \rightarrow Val$
- $r \in V$  is the unique root of the feature structure, i.e., there is exactly one node in  $V$  (which is  $r$ ) such that  $r$  does not have an incoming edge or, to put it differently, there is no  $v \in V, a \in A$  with  $a(v) = r$ .



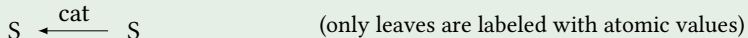
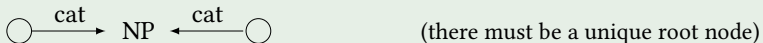
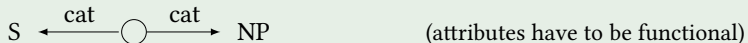
## Attribute-value structures as graphs (3)

### Feature structures as graphs

- possible feature structure:



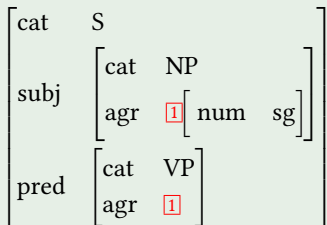
- ill-formed feature structures:



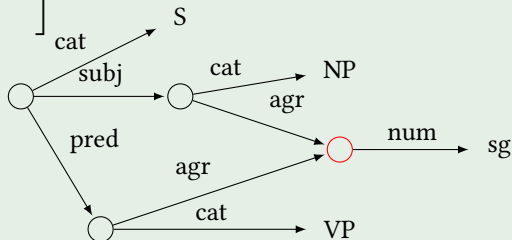
## Attribute-value structures as graphs (4)

Attribute-value graphs are not always trees since we can have more than one incoming edge per node.

### Structure Sharing



structure sharing



## Subsumption and unification (1)

Subsumption: Relation on feature structures:  $S_1$  subsumes  $S_2$  ( $S_1 \sqsubseteq S_2$ ), if  $S_2$  contains (at least) all the information from  $S_1$ .

### Example

Subsumption

Ex.  $S_1$ :  $\left[ \begin{array}{cc} \text{cat} & \text{V} \\ \text{agr} & \left[ \begin{array}{cc} \text{num} & \text{Sg} \end{array} \right] \end{array} \right]$        $S_2$ :  $\left[ \begin{array}{cc} \text{orth} & \text{laughs} \\ \text{cat} & \text{V} \\ \text{agr} & \left[ \begin{array}{cc} \text{pers} & 3 \\ \text{num} & \text{Sg} \end{array} \right] \end{array} \right]$

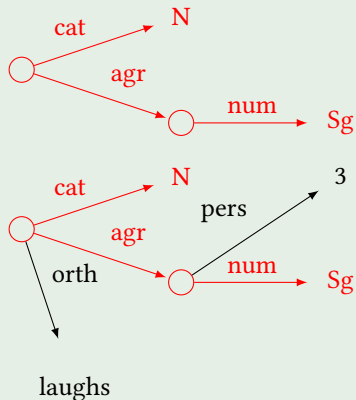
$S_1 \sqsubseteq S_2$

In other words: there is a homomorphism from the nodes of  $S_1$  to the nodes of  $S_2$  that preserves edges and labels and that maps the root of  $S_1$  to the root of  $S_2$ .

## Subsumption and unification (2)

### Example

Subsumption  $S_1$  as a graph and its image under the homomorphism in  $S_2$ :



## Subsumption and unification (3)

### Subsumption

Let  $S_1 = \langle V_1, A, Val, r_1 \rangle$  and  $S_2 = \langle V_2, A, Val, r_2 \rangle$  be feature structures.

$S_1$  subsumes  $S_2$ ,  $S_1 \sqsubseteq S_2$  if there is a function  $h : V_1 \rightarrow V_2$  such that

- $h(r_1) = r_2$ ,
- for all  $v_1, v_2 \in V_1$  and all  $a \in A$ : if  $a(v_1) = v_2$ , then  $a(h(v_1)) = h(v_2)$ , and
- for all  $v \in V_1$  and all  $l \in Val$ : if  $l_{Val}(v) = l$ , then  $l_{Val}(h(v)) = l$ .

## Subsumption and unification (3)

### Subsumption: more examples

$$\blacksquare S_1: \begin{bmatrix} \text{cat} & \text{N} \\ \text{agr} & \begin{bmatrix} \text{num} & \text{Sg} \\ \text{case} & \text{acc} \end{bmatrix} \end{bmatrix} \quad S_2: \begin{bmatrix} \text{orth} & \text{laughs} \\ \text{agr} & \begin{bmatrix} \text{pers} & 3 \\ \text{num} & \text{Sg} \end{bmatrix} \end{bmatrix}$$

$$S_1 \not\sqsubseteq S_2, S_2 \not\sqsubseteq S_1$$

$$\blacksquare S_1: \begin{bmatrix} \text{cat} & \text{N} \\ \text{agr} & \boxed{1} \end{bmatrix} \quad S_2: \begin{bmatrix} \text{cat} & \text{N} \\ \text{agr} & \begin{bmatrix} \text{pers} & 3 \\ \text{num} & \text{Sg} \end{bmatrix} \end{bmatrix}$$

$$S_1 \sqsubseteq S_2$$

## Subsumption and unification (4)

Subsumption is a **partial order**, so it is

- 1 reflexive: each structure subsumes itself  $S \sqsubseteq S$  for all  $S$ ;
- 2 transitive: if  $S_1 \sqsubseteq S_2$  and  $S_2 \sqsubseteq S_3$  then  $S_1 \sqsubseteq S_3$  for all  $S_1, S_2, S_3$ ;
- 3 asymmetric: if  $S_1 \sqsubseteq S_2$  and  $S_2 \sqsubseteq S_1$  then  $S_1 = S_2$ .

An empty feature structure [ ] subsumes all other feature structures.

## Subsumption and unification (5)

A feature structure  $S$  is a **unification** of  $S_1$  and  $S_2$  ( $S_1 \sqcup S_2$ ), if  $S$  is subsumed by both  $S_1$  and  $S_2$  and  $S$  subsumes all other feature structures, that are subsumed by both  $S_1$  and  $S_2$ .

$$\left[ \begin{array}{l} \text{cat} \quad \text{V} \\ \text{agr} \quad \left[ \begin{array}{l} \text{num} \quad \text{Sg} \end{array} \right] \end{array} \right] \sqcup \left[ \begin{array}{l} \text{cat} \quad \text{V} \\ \text{agr} \quad \left[ \begin{array}{l} \text{pers} \quad 3 \end{array} \right] \end{array} \right] = \left[ \begin{array}{l} \text{cat} \quad \text{V} \\ \text{agr} \quad \left[ \begin{array}{l} \text{num} \quad \text{Sg} \\ \text{pers} \quad 3 \end{array} \right] \end{array} \right]$$

To make  $\sqcup$  always defined, we introduce a symbol  $\perp$  that refers to an inconsistent feature structure that is subsumed by all feature structures.

$$\left[ \begin{array}{l} \text{cat} \quad \text{NP} \\ \text{agr} \quad \left[ \begin{array}{l} \text{num} \quad \text{Sg} \end{array} \right] \end{array} \right] \sqcup \left[ \begin{array}{l} \text{cat} \quad \text{V} \\ \text{agr} \quad \left[ \begin{array}{l} \text{num} \quad \text{Sg} \\ \text{pers} \quad 3 \end{array} \right] \end{array} \right] = \perp$$



## Subsumption and unification (6)

Feature structures that are related by the  $\sqsubseteq$  relation, form a **lattice**:  $\sqsubseteq$  is a partial order and for any  $S_1, S_2$  the following holds:

- (sup) There is a feature structure  $S$ , such that  $S_1 \sqsubseteq S$  and  $S_2 \sqsubseteq S$  and  $S$  also subsumes all other feature structures that are subsumed by both  $S_1$  and  $S_2$ .  $S$  is called **Supremum** of  $\{S_1, S_2\}$ .
- (inf) There is a feature structure  $S$ , such that  $S \sqsubseteq S_1$  and  $S \sqsubseteq S_2$  and  $S$  is subsumed by all other structures that subsume both  $S_1$  and  $S_2$ .  $S$  is called **Infimum** of  $\{S_1, S_2\}$ .

From this it follows that with respect to the  $\sqsubseteq$  the smallest element is  $[]$ , and the biggest element is  $\perp$ .

## Typed feature structures (1)

The examples mentioned above implicitly imply that CAT is a syntactic category and AGR is responsible for the agreement. I.e., the following feature structures should not be possible:

$$\left[ \begin{array}{cc} \text{cat} & \text{Sg} \\ \text{agr} & \left[ \begin{array}{cc} \text{num} & 3 \\ \text{pers} & \text{V} \end{array} \right] \end{array} \right] \quad \left[ \text{cat} \left[ \text{agr} \left[ \text{pers} \ 3 \right] \right] \right]$$

However, nothing prevents the existence of such structures so far, as there is no generalisation defined for this case.

Goal: formulate restrictions of the kind “an agreement feature structure can have only attributes NUM, PERS and GEN”.

## Typed feature structures (2)

So we introduce **types** for feature structures:

- Each feature structure has a type  $\tau$ .
- For each type  $\tau$  it is defined which attributes it has and what are the types of the values of these attributes.
- Types are organised in a type hierarchy, where specific types are ordered under the general types.
- Unification operation is extended in order to take care of the types.

## Typed feature structures (3)

Types and their possible arguments are identified using the type hierarchy and attributes for single types.

$$\left[ \begin{array}{l} \textit{agr-structure} \\ \textit{agr} \end{array} \right] \left[ \begin{array}{l} \textit{agr} \\ \textit{num} \quad \textit{num} \\ \textit{gen} \quad \textit{gen} \\ \textit{pers} \quad \textit{pers} \end{array} \right]$$

$$\left[ \begin{array}{l} \textit{determiner} \\ \textit{quant} \end{array} \right] \left[ \begin{array}{l} \textit{noun} \\ \textit{case} \quad \textit{case} \end{array} \right] \left[ \begin{array}{l} \textit{syncat} \\ \textit{cat} \quad \textit{cat} \end{array} \right]$$

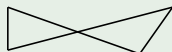
Type *quant*: {every, most, some, none}, Type *num*: {Sg, Pl}, Type *gen*: {m,f,n}, Type *pers*: {1, 2, 3}, Type *case*: {nom, acc, dat}, Type *cat*: {N, V, NP, VP, S, ... }

# Typed feature structures (4)

## Types

$$\begin{bmatrix} agr\text{-}structure \\ agr & agr \end{bmatrix}$$
$$\begin{bmatrix} syncat \\ cat & cat \end{bmatrix}$$
$$\begin{bmatrix} agr \\ num & num \\ gen & gen \\ pers & pers \end{bmatrix}$$
$$\begin{bmatrix} determiner \\ quant & quant \end{bmatrix}$$
$$\begin{bmatrix} noun \\ case & case \end{bmatrix}$$

*agr-structure*      *syncat*



Type hierarchy:

*det*      *noun*

The attributes of *noun* are determined by the types *agr-structure*, *syncat* and *noun*.

$$\begin{bmatrix} noun \\ cat & N \\ case & acc \\ agr & \begin{bmatrix} agr \\ num & Sg \\ gen & m \\ pers & 3 \end{bmatrix} \end{bmatrix}$$

## Extensions (1)

Some linguistic theories use also sets or lists as attribute values.  
Example.: Head-Driven Phrase Structure Grammar (HPSG) codes syntactic trees as feature structures, where all the daughters of the node are provided as a value of the respective attribute in form of a list.

### set attributes

$$\left[ \begin{array}{l} \textit{phrase} \\ \text{dtrs} \left\langle \left[ \begin{array}{ll} \text{cat} & \text{PRO} \\ \text{orth} & \text{I} \end{array} \right], \left[ \begin{array}{ll} \text{cat} & \text{VP} \\ \text{dtrs} & \left\langle \left[ \begin{array}{ll} \text{cat} & \text{V} \\ \text{orth} & \text{love} \end{array} \right], \left[ \begin{array}{ll} \text{cat} & \text{NP} \\ \text{orth} & \text{New York} \end{array} \right] \right\rangle \right] \right\rangle \end{array} \right]$$

## Extensions (2)

- Some systems work directly with feature structures as graphs.
- Some use descriptions of features structures.

Advantage of descriptions: variable expressive power depending on the used Logic (of course in connection with the complexity). Some useful operations:

- 1 Disjunction:  $CASE = acc \vee CASE = dat$
- 2 Negation:  $\neg(CASE = nom)$
- 3 Non-equality of paths:  $SUBJ [CASE] \neq OBJ [CASE]$