A non-identity does not imply a plurality: The Problem of the Many and the semantics of numeral constructions.

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Outline

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The problem of the many (Lewis, 1993/1999; Unger, 1980; Geach, 1962/1980):

A paradox

- Premise: There is exactly one N [in some situtation]
- Conclusion: There are either no Ns or many Ns [in the same situation]

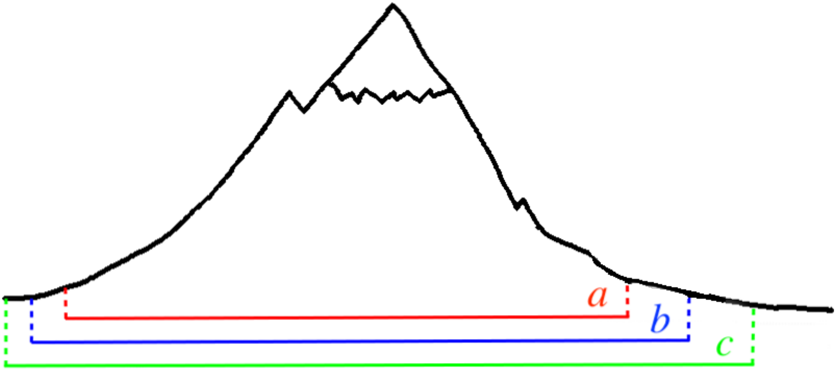
Example:

- Suppose there is a situation in which it is true that there is exactly one cloud in the sky.
- There are multiple, non-identical sums of water droplets, each of which is an equally good candidate to be the extension of that cloud.
- Therefore, either each of these sums of droplets is a cloud and we have many clouds or no sum of droplets is a cloud and so we have no clouds.
- Either way, paradoxically, we do not have one cloud, contrary to our starting assumption.

The problem of the many: clouds

There is one cloud and either $a, b, c \in [\![\text{cloud}]\!]$ or $a, b, c \notin [\![\text{cloud}]\!]$

The problem of the many: mountains



There is one mountain and either $a, b, c \in [\![\text{mountain}]\!]$ or $a, b, c \notin [\![\text{mountain}]\!]$

Properties of the problem of the many

Three properties of the problem

1. Specifically about count nouns
   - # There is exactly one water/mud/air
   - Mass nouns have cumulative extensions (Quine, 1960): two non-identical amounts of water is water.
2. Orthogonal to vagueness in the quantifier
   - None of the examples so far require there to be an unspecified many possible Ns
   - The paradox is generated even if there are only two competing extensions for the N
3. Orthogonal to vagueness in the noun
   - It is tempting to think that the problem of the many is about vagueness (Unger, 1980; McGee and McLaughlin, 2000; Williams, 2006, amongst many others)
   - But there are reasons to doubt this. Two more examples:
     - Houses and the problem of the two (Lewis, 1993/1999)
     - Food processors and the problem of the several

The problem of the two: houses

"You say that a famous architect designed Fred's house; it never crossed your mind to think whether by 'house' you meant something that did or that didn't include the attached garage; neither does some established convention or secret fact decide the issue; no matter, you knew that what you said was true either way." (Lewis, 1993/1999, p. 172)

- Diagnostics for vagueness: borderline cases & blurred boundaries
  - Neither b nor $a \sqcup b$ is a borderline case
  - The inclusion/non-inclusion of the garage is not a blurred boundary
- If *house* is vague, then the vagueness of *house* is not doing any work here

The problem of the several: food processors

A food processor with more attachments no more or less of a food processor with fewer attachments:

- We can have a situation in which there is exactly one food processor, but equally good candidates for the extension.
- This is not about vagueness

The structure of the problem

The problems of the many, two and several have the same struture:

(i) There is exactly one P (for a natural language predicate P, in some specific situation, world, context etc.)

(ii) There is a non-empty, non-singleton set of non-identical entities, $A = \{a_1, ...a_n\}$ such that for all $a_i, a_j \in A$, a_i is a P iff a_j is a P.

(iii) At least one $a_i \in A$ is a P.

(iv) For all $X : X \subseteq A$, and for all $a_i \in X$, if a_i is a P, then there are n Ps, such that $n = |X|$.

(v) Every $a_i \in A$ is a P

(vi) There are n Ps and $n > 1$.

(vii) It is not the case that there is exactly one P.

[Note: For the ”no Ps” outcome, one can deny (iii)]

Spoiler: Deny (iv)

Non-identity does not imply plurality

(i) There is exactly one P (for a natural language predicate P, in some specific situation, world, context etc.)

(ii) There is a non-empty, non-singleton set of non-identical entities, $A = \{a_1, ...a_n\}$ such that for all $a_i, a_j \in A$, a_i is a P iff a_j is a P.

(iii) At least one $a_i \in A$ is a P.

(iv) For all $X : X \subseteq A$, and for all $a_i \in X$, if a_i is a P, then there are n Ps, such that $n = |X|$.

(v) Every $a_i \in A$ is a P

(vi) There are n Ps and $n > 1$.

(vii) It is not the case that there is exactly one P.

I.e. It is possible that $a, b, c \in P$, $a \neq b \neq c$, and we still have only one P

Why is this of interest to semantics?

1. Our theories predict the wrong counting results

- Our standard assumptions about the semantics of (modifier uses of) numeral expressions e.g., *two* in *two mountains* IMPLICITLY assume that non-identity entails plurality.
- So our analyses of *Two mountains*/*clouds*/*houses*/*food processors* give the wrong counting results.

Why is this of interest to semantics?

2. Our theories misclassify count nouns as non-count nouns.

- Problem of the many type cases give us a set of multiple single entities (entities that count as ‘one P’).
- These entities: overlap, stand in proper-parthood relations, and have different sets of atoms depending on how we precisify their extensions.
- These properties are central to the definitions of count nouns in our leading theories.
  - non-overlap (Landman, 2011, 2016)
  - no proper-part relations between members of a set (quantization) (Krifka, 1989; Filip and Sutton, 2017)
  - having a stable set of atoms across precisifications (stable atomicity) (Chierchia, 2010, 2015)
- This means that pretty much all contemporary semantic theories predict that the nouns that can be used to generate the problem of the many are not count nouns.

The plan

Part 1: The details of the problem for semantic theories

- Brief introduction to concepts in classical extensional mereology (CEM)
- Why is the problem of the many a problem for analyses of numeral expressions?
- Why is the problem of the many a problem for theories of count nouns?
  - Disjointness and quantization based theories

Part 2: Theories based upon Weak Quantization

- Weakening the semantic criterion for what makes a noun a count noun.
- Incorporating these results into an analysis of numeral constructions.

Part 3: Back to the problem of the many

- How does this approach compare with alternative proposals?

Outline

Formal assumptions

Classical extensional mereology (see Link, 1983; Krifka, 1986, 1989, amongst many others). Definitions here are taken from (Krifka, 1989)

Mereological sum: $\sqcup$

- $a \sqcup b$ is the sum of *a* and *b*, it is an entity just as *a* and *b* are
  - Warning: sums do not have to be individuals
- Complete: $\forall P.\forall x, y[P(x) \wedge P(y) \rightarrow \exists z\ [z = x \sqcup y]]$
- Commutative: $\forall x, y[x \sqcup y = y \sqcup x]$
- Idempotent: $\forall x[x \sqcup x = x]$
- Associative: $\forall x, y, z\ [(x \sqcup y) \sqcup z = x \sqcup (y \sqcup z)]$

Part: $\sqsubseteq$

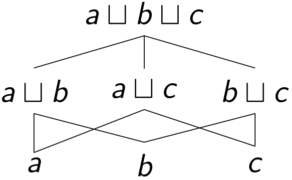
- $\forall x, y\ [x \sqsubseteq y \leftrightarrow x \sqcup y = y]$

Proper Part: $\sqsubset$

- $\forall x, y\ [x \sqsubset y \leftrightarrow x \sqsubseteq y \wedge x \neq y]$

Algebraic semilattices

Semilattice structures (minus the bottom element) can be *generated* from entities under sum:



This means we go from domains of individuals $\mathcal{D}_e = \{a, b, c\}$, to:

$$\mathcal{D}_e = \left\{ \begin{array}{ccc} & a \sqcup b \sqcup c, & \\ a \sqcup b, & a \sqcup c, & b \sqcup c, \\ a, & b, & c, \end{array} \right\}$$

Outline

Numeral expressions and cardinality functions

Standard assumption: The semantics of numeral expressions is based on a CARDINALITY FUNCTION applied to a part-set.

$$\forall x.\forall P.\ \mu_{\#}(x, P) = |\{y : y \sqsubseteq x, P(y)\}|$$

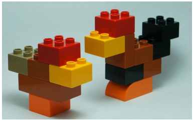
The cardinality of x with respect to P is the cardinality of the set of P-parts of x.

Example: $P = \{a, b, c\}$

$$\begin{aligned} \mu_{\#}(a, P) &= |\{a\}| &&= 1 \\ \mu_{\#}(a \sqcup c, P) &= |\{a, c\}| &&= 2 \\ \mu_{\#}(a \sqcup b \sqcup c, P) &= |\{a, b, c\}| &&= 3 \end{aligned}$$

Why do we need indexing to P?

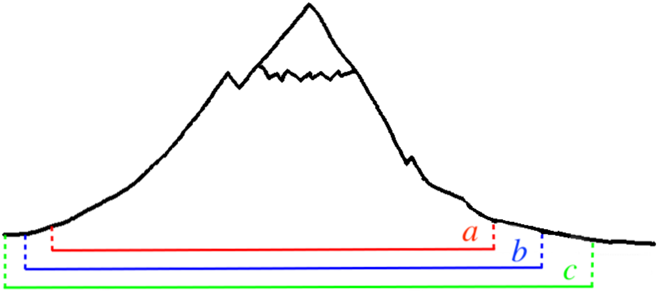
- The same entity can have different cardinalities relative to different predicates (Link, 1983)



= 2 model birds or 15 bricks

The problem for numeral expressions

The problem of the many seems to force us to say that either all of *a*, *b* and *c* are in the extension of *mountain* or none are.



If none are then there is no mountain:

$$a, b, c \notin mountain \rightarrow \mu_{\#}(a \sqcup b \sqcup c, mountain) = 0$$

If all are then there are at least three mountains:

$$a, b, c \in mountain \rightarrow \mu_{\#}(a \sqcup b \sqcup c, mountain) \geq 3$$

Diagnosis

We assumed in the meaning of $\mu_{\#}$ a set theoretic cardinality based on part set:

$$|\{y : y \sqsubseteq x, P(y)\}|$$

- What idealisations does the simple 'cardinality of a part-set' assumption make?
  - Since $\{a, a\} = \{a\}$, it follows that $|\{a, a\}| = 1$
- Therefore, our definition implicitly assumes that any two non-identical entities in P will count as two Ps.
- But this is precisely one of the premises in the paradox!
- First indication about what the 'problem premise' is.

$\Rightarrow$ We should amend our definition for $\mu_{\#}$

Outline

Overview

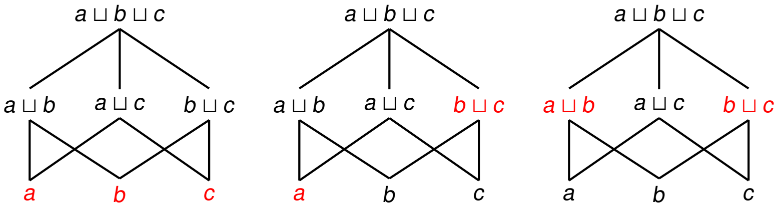
- Two theories of count nouns: disjointness-based and quantization-based
  - I can discuss stable atomicity (Chierchia, 2010) in the Q&A if anyone is interested.
- All disjoint predicates are quantized predicates (disjointness is logically stronger than quantization)
- The problem of the many shows that quantization-based theories are too strong.
- Disjointness based theories are therefore also too strong.
- Therefore, we need a weaker theory.

Theory 1: Count predicates specify disjoint sets.

Overlap, $\circ$, is defined: $\forall x, y\ [x \circ y \leftrightarrow \exists z\ [z \sqsubseteq x \wedge z \sqsubseteq y]]$

$$\forall P[\mathcal{D}(P) \leftrightarrow \forall x \in P. \forall y \in P[x \neq y \rightarrow \neg(x \circ y)]] \quad \text{(Disjoint, } \mathcal{D}\text{)}$$

P is disjoint iff no two entities in the extension of *P* overlap



$\mathcal{D}(\{a, b, c\})$ $\mathcal{D}(\{a, b \sqcup c\})$ $\neg\mathcal{D}(\{a \sqcup b, b \sqcup c\})$

Theory 1: Count predicates specify disjoint sets.

- Explicitly disjointness-based theories:
  - Landman (2011, 2016); Sutton and Filip (2017, 2016b); de Vries and Tsoulas (2018)
- Theories which stress disjointness as one of a set of important properties:
  - Rothstein (2010, 2017); Chierchia (2010)

Most stress the important of context sensitivity/the availability of different schemas of individuation.

Theory schema:

$[_{\text{N}}\ \text{X}]$ is count iff $\forall c.\mathcal{D}(B(\llbracket \text{X} \rrbracket^{c}))$ (Count definition for $\mathcal{D}$)

The counting base for $\llbracket \text{X} \rrbracket$ is *disjoint* at every context

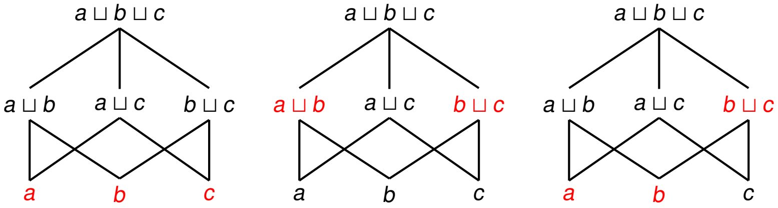
Where, e.g. $B(\llbracket \text{cats} \rrbracket^{c}) = B(\llbracket \text{cat} \rrbracket^{c}) = \llbracket \text{cat} \rrbracket^{c}$

- The entities that count as 'one'

Theory 2: Count predicates specify quantized sets

$$\forall P[\mathcal{Q}(P) \leftrightarrow \forall x \in P.\forall y \in P[\neg(x \sqsubset y)]] \qquad \text{(Quantized, } \mathcal{Q})$$

P is quantized iff no two entities in the extension of *P* are proper parts of each other.



$\mathcal{Q}(\{a, b, c\})$ $\qquad$ $\mathcal{Q}(\{a \sqcup b, b \sqcup c\})$ $\qquad$ $\neg\mathcal{Q}(\{a, b, b \sqcup c\})$

Theory 2: Count predicates specify quantized sets

Originally proposed by:

- Bach (1981) (*antisubdivisible*)
- Krifka (1986, 1989)

More recently, made explicitly context/individuation schema-sensitive

- Filip and Sutton (2017)

Theory schema:

$[_{\text{N}}\ \text{X}]$ is count iff $\forall c.\mathcal{Q}(B(\llbracket \text{X} \rrbracket^{c}))$ (Count definition for $\mathcal{Q}$)

The counting base for $\llbracket \text{X} \rrbracket$ is *quantized* at every context

Where, e.g. $B(\llbracket \text{cats} \rrbracket^{c}) = B(\llbracket \text{cat} \rrbracket^{c}) = \llbracket \text{cat} \rrbracket^{c}$

- The entities that count as 'one'

Disjointness and Quantization: Stopping counting from ‘going wrong’

Specifying Disjointness (Landman, 2011) or Quantization (Filip and Sutton, 2017) as a criterion for counting is based on stopping counting from going wrong:

> *My proposal is formulated in terms of overspecification: I propose that when you look down in a mass denotation you see too many building blocks. And hence, when you count building blocks in a mass denotation, you will count them wrong. (Landman, 2011, p.17)*

Implicit assumption: Overlapping, non-identical entities count as more than one.
Grammatical counting requires resolving overlap/non-quantization relative to context

- Second indication about what the problem premise is
  - A too tight connection between non-identity and a plurality of entities

Disjointness and Quantization: Implicational relations

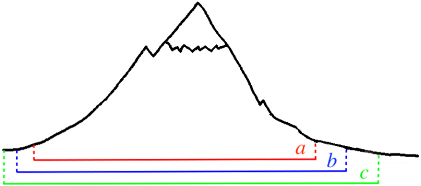
Quantization $\mathcal{Q}$ is logically weaker than Disjointness $\mathcal{D}$:

- $\mathcal{D}(P) \vDash \mathcal{Q}(P)$
- I.e., all disjoint predicate are quantized

Implication: If the Problem of the Many is problematic for a quantization-based theory, it is also problematic for a disjointness based theory.

The problem for Quantization theories

Assuming that at least one of a, b and c is in the extension of *mountain*, then all of a, b, and c are (the problem of the many):



- Mountain-like cases are counter-examples to quantized ($\mathcal{Q}$) theories.
- The set of $\{a, b, c\}$ is not quantized (since $a \sqsubset b$ and $b \sqsubset c$).

Assuming that both e and $d \sqcup e$ is in the extension of house, (the problem of the two):

- House-like cases are counter-examples to quantization-based ($\mathcal{Q}$) theories.
- The set of $\{e, d \sqcup e\}$ is not quantized (since $e \sqsubset d \sqcup e$).

The problem for theories of countability

Nouns like *mountain* and *house* do not denote quantized sets

- A semantic criterion fo count Ns should not exclude nouns such as *mountain*

The problem is just as severe for disjointness-based theories

- $\mathcal{D}(P) \vDash \mathcal{Q}(P)$

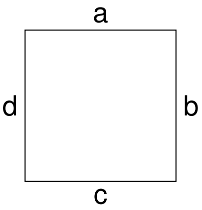
Outline

Context-sensitivity

Fences:

Nouns such as the following have counting criteria that vary with context (Rothstein, 2010; Zucchi and White, 1996, 2001, a.o.)

- *fence*, *wall*, *hedge*, *sequence*, *twig*, *branch*



$\{a, b, c, d\} \subseteq [\![\text{fence}]\!]^{c_{4f}}$ (a *4-fence* context)

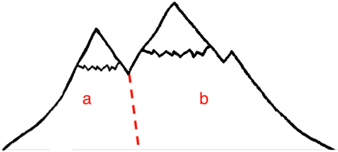
$\{a \sqcup b, c \sqcup d\} \subseteq [\![\text{fence}]\!]^{c_{2f_i}}$ (a *2-fence* context)

$\{a \sqcup b \sqcup c \sqcup d\} \subseteq [\![\text{fence}]\!]^{c_{1f}}$ (a *1-fence* context)

... ...

Mountains:

There are at least two different types of contexts (ways of individuating) the mountain below (example from Chierchia, 2010):



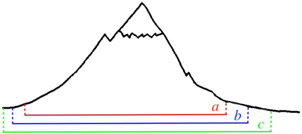
$a \sqcup b \in [\![\text{mountain}]\!]^{c_1}$ (1 mountain)

$a, b \in [\![\text{mountain}]\!]^{c_2}$ (2 mountains)

A proliferation of contexts does not help

Problem of the many examples present a different case:

- If any one of *a*, *b* and *c* are a mountain, relative to the SAME context, then all are.



Possible fix: Allow fine-grained contexts in our quantization theory?

- E.g., For every millimetre difference in the boundary of a mountain, there is a distinct context that selects that boundary as *the* extension of the mountain.

Implausible:

- An over-population of contexts - more contexts than there are ways to draw a boundary around a mountain!
- Not what contexts are meant to do.

Constraining the Quantized criterion

Constraint: Count nouns can specify non-quantized sets for counting if the ways of resolving failures of quantization do not change cardinality

A 'permitted' violation of quantization: $|\{a \sqcup b, a\}| = 2$

- We can 'resolve' this in two ways:
  - $|\{a \sqcup b\}| = 1$
  - $|\{b\}| = 1$
- Both have the same cardinality

A 'non-permitted' violation of quantization: $|\{a \sqcup b, a, b\}| = 3$

- We can 'resolve' this in two ways:
  - $|\{a \sqcup b\}| = 1$
  - $|\{a, b\}| = 2$
- A change in cardinality, so not allowed

Problem:

- This effectively weakens the Q to something else [it concedes that Quantization is too strong]
- So why not explicitly weaken Quantization instead of hand-writing an exception?

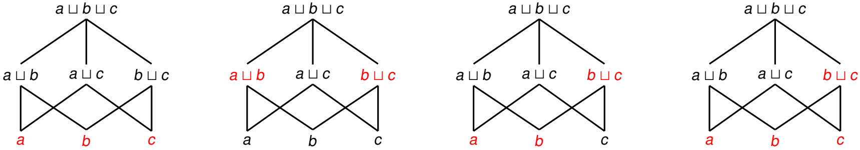
Outline

- The problem of the many
- Brief overview of classical extensional mereology
- The problem of the many and numeral expressions
- The problem of the many and the count/mass distinction
  - Attempts to save a quantization-based theory
- Weak quantization as a count criterion
- Weak quantization, and numeral expressions
- Comparison with other solutions

A new alternative: Count predicates specify *weakly* quantized sets

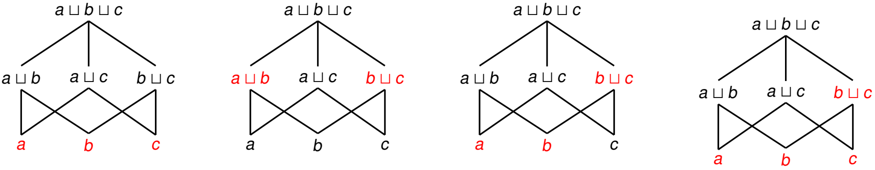
$$\forall P[\mathcal{U}(P) \quad \leftrightarrow \quad \forall x.\forall y.\forall z[(P(x) \wedge y \sqsubseteq x \wedge z \sqsubseteq x \wedge \neg y \circ z) \\ \rightarrow \neg(P(y) \wedge P(z))]] \qquad \text{(Weakly quantized, } \mathcal{U})$$

P is *weakly quantized* iff at most one of two non-overlapping parts of a *P* is a *P*.



$\mathcal{U}(\{a, b, c\})$ $\qquad$ $\mathcal{U}(\{a \sqcup b, b \sqcup c\})$ $\qquad$ $\mathcal{U}(\{a, b, b \sqcup c\})$ $\qquad$ $\neg\mathcal{U}(\{a, b, c, b \sqcup c\})$

Contrasting the criteria



$$\mathcal{D}(\{a, b, c\})$$
$$\mathcal{Q}(\{a, b, c\})$$
$$\mathcal{U}(\{a, b, c\})$$

$$\neg\mathcal{D}\{a \sqcup b, b \sqcup c\})$$
$$\mathcal{Q}(\{a \sqcup b, b \sqcup c\})$$
$$\mathcal{U}(\{a \sqcup b, b \sqcup c\})$$

$$\neg\mathcal{D}(\{a, b, b \sqcup c\})$$
$$\neg\mathcal{Q}(\{a, b, b \sqcup c\})$$
$$\mathcal{U}(\{a, b, b \sqcup c\})$$

$$\neg\mathcal{D}(\{a, b, c, b \sqcup c\})$$
$$\neg\mathcal{Q}(\{a, b, c, b \sqcup c\})$$
$$\neg\mathcal{U}(\{a, b, c, b \sqcup c\})$$

A new alternative: Count predicates specify *weakly* quantized sets

$[_{\text{N}}$ X$]$ is count iff $\forall c.\mathcal{U}(B(\llbracket \text{X} \rrbracket^{c}))$ (Count definition for $\mathcal{U}$)

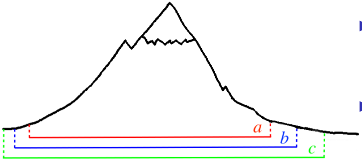
The counting base for $\llbracket \text{X} \rrbracket$ is *weakly quantized* at every context

Where, e.g. $B(\llbracket \text{cats} \rrbracket^{c}) = B(\llbracket \text{cat} \rrbracket^{c}) = \llbracket \text{cat} \rrbracket^{c}$

- The entities that count as 'one'

- Maintaining much of the recent work and developments on the count/mass distinction
  - E.g., we still maintain a central role for the context sensitivity of individuation in order to deal with *fence*-like cases
- We have just slightly weakened the extensional mereological property underpinning the theory

The problem of the many and weak quantization as a criterion for count nouns



No problem!

- The set of $\{a, b, c\}$ is weakly quantized
- Although $a \sqsubset b$ and $b \sqsubset c$, at most one of any two non-overlapping proper parts is a mountain.

No problem!

- The set of $\{e, d \sqcup e\}$ is weakly quantized
- Although $e \sqsubset d \sqcup e$, there are no two no overlapping subparts of the house that are both houses.

One further motivation for weak quantization

Countability in the verbal domain:

(1) John walked for an hour/#in an hour. (atelic VP)

(2) John walked to the shop in an hour/#for an hour. (telic VP)

Intuitive idea:

- The VP in (1) is atelic because the paths of walking events are unbounded
- The VP in (2) is telic because the paths of walking-to-the-shop events are unbounded

Problem (acknowledged by Krifka):

- Walking-to-the-shop event paths are not quantized:

$$\begin{array}{r} a \to b \to c \to d \\ b \to c \to d \\ c \to d \end{array}$$

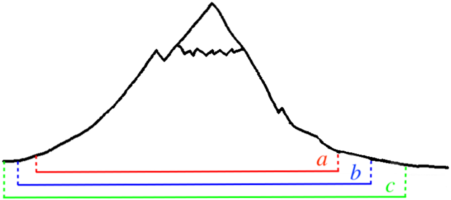
Solved by weak quantization!

Maximization?

Maximization as a reaction to failures of (a) Quantization or to (b) the Problem of the many:

(a) (Filip and Rothstein, 2005; Zucchi and White, 2001)
(b) (Burke, 1994; Sider, 2001)
   - A countable set contains only the maximal elements in the context

- E.g., only *c* would count as the mountain



“Very large proper parts of houses, tables and chairs, rocks and mountains, persons and cats, are not themselves houses, tables, chairs, rocks, mountains, persons or cats.” (Sider, 2001, p. 357)

Worries about maximization

(i) There is exactly one P (for a natural language predicate P, in some specific situation, world, context etc.)

(ii) There is a non-empty, non-singleton set of non-identical entities, $A = \{a_1, ...a_n\}$ such that for all $a_i, a_j \in A$, a_i is a P iff a_j is a P.

(iii) At least one $a_i \in A$ is a P.

(iv) For all $X : X \subseteq A$, and for all $a_i \in X$, if a_i is a P, then there are n Ps, such that $n = |X|$.

(v) Every $a_i \in A$ is a P

(vi) There are n Ps and $n > 1$.

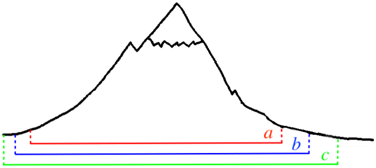
(vii) It is not the case that there is exactly one P.

Amounts to a denial of the 'equally good candidates' assumption

- Why should the maximal entity be privileged?
- Implies e.g. that the house-minus-garage is not a house.

Outline

The problem for cardinality functions remains



$\{a, b, c\} \subseteq [\![\text{mountain}]\!]^c$

- But there's still only one mountain.

Standard cardinality function:

$$\mu_{\#}(x, P) = |\{y : y \sqsubseteq x, P(y)\}|$$

But this was problematic work, since:

$$\mu_{\#}(a \sqcup b \sqcup c, [\![\text{mountain}]\!]^c) = 3$$

The wrong result!

Weakly quantized with respect to P

What we have so far: Weakly quantized, $\mathcal{U}$:

$$\forall P[\mathcal{U}(P) \leftrightarrow \forall x.\forall y.\forall z[(P(x) \wedge y \sqsubseteq x \wedge z \sqsubseteq x \wedge \neg y \circ z) \rightarrow \neg(P(y) \wedge P(z))]]$$

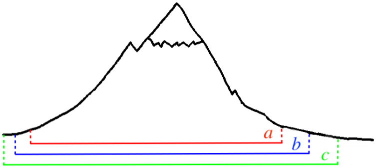
P is *weakly quantized* iff at most one of any two non-overlapping parts of a P is a P.

What we need is the notion of a weakly quantized entitity relative to a predicate:

$$\forall x.\forall P[\mathfrak{U}(x, P) \leftrightarrow \forall y.\forall z[(y \sqsubseteq x \wedge z \sqsubseteq x \wedge \neg y \circ z) \rightarrow \neg(P(y) \wedge P(z))]]$$

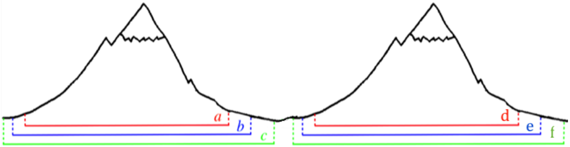
(Weakly quantized x with respect to P, $\mathfrak{U}(x, P)$)

In other words, the set of P parts of x is weakly quantized



Example: $\mathfrak{U}(a \sqcup b \sqcup c, [\![\text{mountain}]\!])$

Counting based on weak quantization



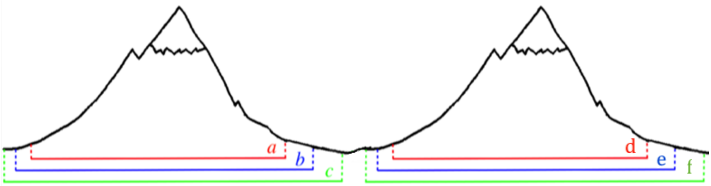
We want to have:

$$\mu_{\#}(a \sqcup b \sqcup c \sqcup d \sqcup e \sqcup f, [\![\text{mountain}]\!]^c) = 2$$

Proposal:

- Weak quantization (relative to a context) tell us what counts as 1 relative to a predicate, and so a sum of weakly quantized entities that is not weakly quantized relative to that predicate counts as more than one.
- We can count any two entities that are mountains as two mountains if their sum is NOT weakly quantized with respect to $[\![\text{mountain}]\!]$

Counting based on weak quantization



- I.e. the sum of any two elements in the set $\{a, b, c, d, e, f\}$ count as two mountains iff they can be split into two non-overlapping parts, each of which counts as a mountain.

Example 1:

$c \sqcup f$ counts as two mountains because $c, f \in [\![\text{mountain}]\!]$ and $\neg\mathfrak{U}(a \sqcup f, [\![\text{mountain}]\!])$

Example 2:

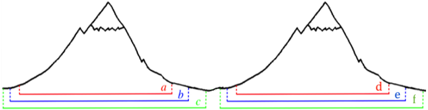
$d \sqcup f$ does not count as two mountains because, although $d, f \in [\![\text{mountain}]\!]$, $\mathfrak{U}(d \sqcup f, [\![\text{mountain}]\!])$

Formalising the counting criterion

$$\mu_{\#}(x, P) = n \leftrightarrow \quad \exists Q.|Q| = n \wedge \ Q \subseteq P \wedge \sqcup Q = x \wedge \\ \forall y, z \in Q[y \sqsubseteq x \wedge z \sqsubseteq x \wedge \neg \mathfrak{U}(y \sqcup z, P)]$$

In words: x has a cardinality of n with respect to P iff there is a partition of x into n parts such that the sum of any two parts is not weakly quantized with respect to P

Example



$$\mu_{\#}(x,P) = n \leftrightarrow \quad \exists Q.|Q| = n \wedge \ Q \subseteq P \wedge \sqcup Q = x \ \wedge$$
$$\forall y,z \in Q[y \sqsubseteq x \wedge z \sqsubseteq x \wedge \neg \mathfrak{U}(y \sqcup z, P)]$$

$$\mu_{\#}(a \sqcup b \sqcup c \sqcup d \sqcup e \sqcup f, [\![\text{mountain}]\!]^c) \quad = 2$$

Why?

- There is a way of splitting $a \sqcup b \sqcup c \sqcup d \sqcup e \sqcup f$ into two parts
  - $a \sqcup b \sqcup c$ and $d \sqcup e \sqcup f$
  - Such that $Q = \{a \sqcup b \sqcup c, d \sqcup e \sqcup f\}$ and $|Q| = 2$
  - $a \sqcup b \sqcup c, d \sqcup e \sqcup f \in [\![\text{mountain}]\!]^c$, a weakly quantized set.
- But $a \sqcup b \sqcup c \sqcup d \sqcup e \sqcup f$ is NOT weakly quantized with respect to $[\![\text{mountain}]\!]^c$.
- And, there is no way of splitting $a \sqcup b \sqcup c \sqcup d \sqcup e \sqcup f$ into any other number of parts that satisfies these conditions for a different cardinality.

Taking stock

A new, weaker condition for when a noun is count:

- It specifies a weakly quantized set of entities (relative to context)

A new, proposal for the cardinality functions that underpin numeral semantics:

- Basic idea: set cardinality is not sufficient
- If something is n Ps, then it can be split into n non-overlapping parts, each of which is a P

This means that we can have a sum of entities, each of which is a P, but where that sum only counts as one P, since it cannot be split into two non-overlapping parts, each of which is a P.

- And thereby a solution to the problem of the many.
- It is false that: For all $X : X \subseteq A$, and for all $a_i \in X$, if a_i is a P, then there are n Ps, such that $n = |X|$.

Outline

Comparisons – Lewis and partial identity

"concedes that the many are [mountains] but seeks to deny that the [mountains] are really many." (Lewis, 1993/1999, p. 175)

Lewis's initial proposal: partial identity

- The many are many (strictly), but the many are almost identical (substantial overlap)

My view is different, as can be seen by Lewis' example that is problematic for his view:

- *house* and *house* $\sqcup$ *garage* are equally good candidates to be the house
- No significant overlap
- But $\mathcal{U}(\{\textit{house}, \textit{house} \sqcup \textit{garage}\})$

Lewis' refined proposal: a supervaluationism/partial identity hybrid.

- A weak quantization based theory has no need for hybrids

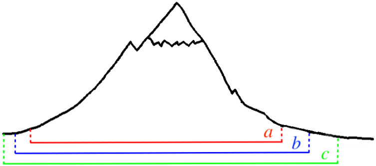
Comparisons cont. – Geach and relative identity

More like Geach's (1962/1980) relative identity?

> *"The price we have to pay is that we must regard "—is the same [P] as—" as expressing only a certain equivalence relation, not an absolute identity restricted to [Ps]" (p. 216)*

- So, this allows for *a* to be the same mountain as *b*, even if, for some *P*, *a* is not the same *P* as *b*.

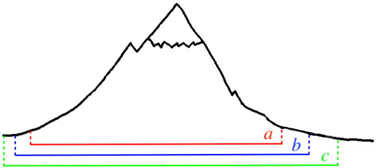
Inconsistent with Leibniz's principle of the indiscernibility of identicals – A reductio of the view (Burke, 1994)?



Suppose that *b* is 2km across: *a* is the same mountain as *c*, but *a* has the property of being less than 2km across, whereas *c* does not.

⇒ The mountain both is and is not less than 2km across

My account is not a relative identity account



So far, I have argued that:

Different mountain candidates are in the denotation of the singular count N, *mountain*:

$$\{a, b, c\} \in [\![\text{mountain}]\!]^c$$

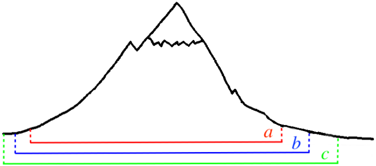
But, the sum of these candidates counts as only one mountain:

$$\mu_{\#}(a \sqcup b \sqcup c, [\![\text{mountain}]\!]^c) = 1$$

I have not made any claims about e.g., when *x is the same mountain as y*

- Do I owe an account of this?

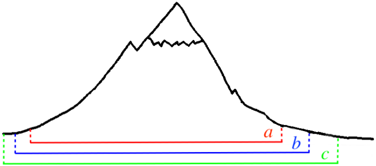
What to say about same-saying?



Option 1: One *P* formed of different *P*s

- *a* and *c* are not the same mountain, but the sum of *a* and *c* counts as one mountain
  - \+ No break of Leibniz' principle of the discernibility of identicals.
  - \+ Is the mountain more or less than 2km across?
    - That depends how we demarcate the mountain...
  - – Tension between there being *different mountains* and yet *only one mountain*

What to say about same-saying?



Option 2: Deny that *the same ... as ...* always encodes identity

- Ambiguity between "=" and whether the sum counts as 'one'
- Suppose that my cup that was whole at t_1 got chipped at t_2. Is it the same cup?
  - No - now it's chipped
  - Yes - the sum of the two is still just one cup
- *a* and *c* are not the same mountain in one sense, but they are the same mountain in another.
  - \+ No break of Leibniz' principle of the discernibility of identicals.
    – That concerns only "="
  - \+ Is the mountain more or less than 2km across?
    - That depends how we demarcate the mountain...
  - \+ No tension between *two different mountains* and *only one mountain*

Summary and Conclusions

- In most circumstances, counting based on set-theoretic cardinality functions is a reliable heuristic.
- However, it is an oversimplification
  - A non-identity does not always mean a plurality
- The problem of the many plays exactly upon the cases where our heuristic fails.
- We can conservatively adjust our semantic models accordingly (most of the work was already done by others) and this approach gives us a way out of the problem of the many

Thank you very much for listening!

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