Parsing

Homework 2 (CFG, PDA), due 19 April 2022 before the course

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Question 1 (CFG)

1. Consider the CFG G_1 with non-terminals $\{S, A, B\}$, terminals $\{a, b, c\}$, start symbol S and productions

 $S \to ASBB \,|\, ASB \,|\, c \quad A \to aA \,|\, a \quad B \to b$

Transform G_1 into an equivalent CFG G'_1 in Chomsky Normal Form.

2. Consider the CFG G_2 with non-terminals $\{S, T\}$, terminals $\{a, b, c, e\}$, start symbol S and productions

 $S \to Tb \quad S \to a \quad T \to Sa$

Transform G_2 into an equivalent CFG G'_2 without left recursion. Do so with the algorithm from the course. Perform two transformation, one with S having index 1, T index 2, and the other one with the reverse indexing. Which grammars do you obtain?

Make sure that you remove useless symbols after each transformation step (if there are any).

Solution:

- 1. $S \rightarrow AX \mid AY \mid c \quad X \rightarrow SZ \quad Y \rightarrow SB \quad Z \rightarrow BB \quad A \rightarrow C_aA \mid a \quad C_a \rightarrow a \quad B \rightarrow b$
- 2. (a) First indexing:

 $S_1 \rightarrow T_2 b$ $S_1 \rightarrow a$ $T_2 \rightarrow S_1 a$ I Replace $T_2 \rightarrow S_1 a$ with $T_2 \rightarrow T_2 ba$ and $T_2 \rightarrow aa$ \rightarrow new set of productions: $S_1 \rightarrow T_2 b$ $S_1 \rightarrow a$ $T_2 \rightarrow T_2 ba$ $T_2 \rightarrow aa$ II Take a new non-terminal X and replace the T_2 -productions with $T_2 \rightarrow aa \mid aaX$ and $X \rightarrow baX \mid ba$ \rightarrow new set of productions: $S_1 \rightarrow T_2 b \mid a$ $T_2 \rightarrow aa \mid aaX$ $X \rightarrow baX \mid ba$

(b) Second indexing:

 $\begin{array}{lll} S_2 \rightarrow T_1 b & S_2 \rightarrow a & T_1 \rightarrow S_2 a \\ \text{I Replace } S_2 \rightarrow T_1 b \text{ with } S_2 \rightarrow S_2 a b \\ \sim \text{ new set of productions: } & S_2 \rightarrow S_2 a b & S_2 \rightarrow a & T_1 \rightarrow S_2 a \\ \text{Now, } T_1 \text{ has become useless and can be removed.} \\ \sim \text{ new set of productions: } & S_2 \rightarrow S_2 a b & S_2 \rightarrow a \\ \text{II Take a new non-terminal } X \text{ and replace the } S_2 \text{ productions with } S_2 \rightarrow a \mid a X \text{ and } X \rightarrow a b X \mid a b \\ \sim \text{ new set of productions: } & S_2 \rightarrow a \mid a X & X \rightarrow a b X \mid a b \end{array}$

Question 2 (PDA)

Give a PDA that recognizes the following language, acceptance with final state: $\{(ab)^n b^m \mid m \ge n \ge 0\}$.

Solution: Initial stack symbol is #.



Question 3 (PDA)

Consider the following PDA M, initial stack symbol is #.



- 1. Which of the following words are elements of L(M)?(i) a(ii) abab(iii) babba(iv) babbaa(v) ε
- 2. Give L(M) and N(M).

Solution:

- 1. (ii) and (iv)
- 2. $L(M) = N(M) = \{w \in \{a, b\}^+ \mid |w|_a = |w|_b\}$