Parsing Beyond CFG
Homework 9: EPDA
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Question 1

Give an EPDA M that recognizes the language $\{wf(w)|w \in \{a,b\}^*, f$ a homomorphism with $f(a) = c, f(b) = d\}$ with an empty stack.

Hint: When having symbols A_1 A_2 A_3 on the top stack (A_3 being the topmost), and you want to access the bottom symbol first, you can, with appropriate transitions, remove the symbols stepwise from this stack and put them in new stacks below the topmost stack. I.e., $\ddagger A_1 A_2 A_3$ becomes $\ddagger A_3 \ddagger A_3 \ddagger A_1$. (This is something, PDAs cannot do, you need a second stack (here the stack of stacks) for that.)

Solution:
$M = \langle \{q_0, q_1, q_2, q_3\}, \{a, b\}, \{\#, A, B\}, \delta, q_0, \{q_3\}, \# \rangle$ mit
$\delta(q_0, \epsilon, \#) = \{(q_3, \epsilon, \epsilon, \epsilon), (q_1, \epsilon, \#, \epsilon)\}$
$\delta(q_1, a, X) = \{(q_1, \epsilon, XA, \epsilon)\}$ for all $X \in \{A, B, \#\}$
$\delta(q_1, b, X) = \{(q_1, \epsilon, XB, \epsilon)\}$ for all $X \in \{A, B, \#\}$
$\delta(q_1, \epsilon, X) = \{(q_2, \epsilon, X, \epsilon)\}$ for all $X \in \{A, B, \#\}$
$\delta(q_2, \epsilon, A) = \{(q_2, \ddagger A, \epsilon, \epsilon)\}$
$\delta(q_2, \epsilon, B) = \{(q_2, \ddagger B, \epsilon, \epsilon)\}$
$\delta(q_2, \epsilon, \#) = \{(q_3, \epsilon, \epsilon, \epsilon)\}$
$\delta(q_3, c, A) = \{(q_3, \epsilon, \epsilon, \epsilon)\}$
$\delta(q_3, d, B) = \{(q_3, \epsilon, \epsilon, \epsilon)\}$

Question 2 Consider the following EPDA M:

$M = \langle \{q_0, q_1, q_2\}, \{a, b, c, d\}, \{\#, B, C, D\}, \delta, q_0, \{q_2\}, \# \rangle$ mit
$\delta(q_0, a, \#) = \{(q_0, \epsilon, B, \epsilon)\}$
$\delta(q_0, a, B) = \{(q_0, \epsilon, BB, \epsilon), (q_1, \epsilon, B, \epsilon)\}$
$\delta(q_1, b, B) = \{(q_1, \ddagger C, \epsilon, \epsilon)\}$
$\delta(q_1, c, C) = \{(q_2, \epsilon, D, \epsilon)\}$
$\delta(q_2, d, D) = \{(q_2, \epsilon, \epsilon, \epsilon)\}$
$\delta(q_2, c, C) = \{(q_2, \epsilon, D, \epsilon)\}$

1. Which language is the language $N(M)$?

2. Give the trace, i.e., the sequence of configurations (of tuples of state, stack, already processed input, remaining input) that one obtains for the input word $w = aabcd$. **List only successful configurations.**

Solution:

1. $L_3 = \{a^{n+1}b^n(cd)^n|n \geq 1\}$

2. $(q_0, \ddagger\#, \epsilon, aabcd)$
   $\vdash (q_0, \ddagger B, a, abcd)$
   $\vdash (q_1, \ddagger B, aa, bcd)$
   $\vdash (q_1, \ddagger C, aab, cd)$
   $\vdash (q_2, \ddagger D, aabc, d)$
   $\vdash (q_2, \epsilon, aabcd, \epsilon)$