In: Proceedings of the Twelfth Amsterdam Colloquium, Amsterdam, December 18–21, 1999, pp. 199–204

Semantics for Attribute-Value Theories

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Summary: Attribute-value (AV) descriptions are reconstructed from natural language by regimentation and formalization within first-order predicate logic. The introduction of appropriate predicate operators then leads to AV expressions of the usual Kasper-Rounds type. We present a slight extension which permits relations between attribute values. A straightforward modification of standard AV logic turns out to be sound and complete with respect to first-order derivability granted that attributes are functional. Demonstrating this is part of our second concern which is to apply geometric logic and locale theory to AV theories like HPSG. Viewing AV theories as propositional geometric theories provides a crisp characterization of the denotation of an AV theory as the point space of its classifying locale.

Attribute-Value Descriptions

The objective of attribute-value theories is to characterize the entities of the domain in question by characterizing their *attribute values* as being of a certain type or bearing certain relations to each other. Typical attribute values, say, of a person are its mother and its birthplace. *Attributes* thus are (dyadic) functional relations like that of mother to child or of place to offspring.

Natural language counterparts of AV descriptions are predicates of the form 'someone whose father is a plumber', 'someone whose wife hates his mother', or 'someone whose father is her employer'. The first predicate can be paraphrased by the regimented version 'x such that the father of x is a plumber', using variables as formalized pronouns. Writing '$\{x \mid \ldots x \ldots\}$' for '$x$ such that $\ldots x \ldots$', it can be rendered into '$\{x \mid P(\iota y Fyx)\}$', with '$F$' for '$\{xy \mid x$ is father of $y\}$' and 'P' for 'plumber'. Elimination of the definite description leads to:

$$\{x \mid \exists y(Fyx \wedge Py) \;\wedge\; \forall yz(Fyx \wedge Fzx \rightarrow y = z)\}\,.$$

Presupposing 'F' as functional, that is,

$$\forall xyz(Fxz \wedge Fyz \rightarrow x = y)\,,$$ [1]

the foregoing predicate becomes equivalent to the *inverse image* or *Pierce product* '$\{x \mid \exists y(Fyx \wedge Py)\}$' of '$P$' by '$F$', abbreviated by '$F{:}P$'.

Polyadic predication as in the second example can be handled analogously, which leads to '$\{x \mid H(\iota y Fyx, \iota y Gyx)\}$' and further to '$\{x \mid \exists yz(Fyx \wedge Gzx \wedge Hyz)\}$', where '$F$' and '$G$' are again assumed to be functional and 'H' stands for '$\{xy \mid x$ hates $y\}$'. With '$\langle F_1, \ldots, F_n\rangle$' in place of:

$$\{y_1 \ldots y_n x \mid F_1 y_1 x \wedge \ldots \wedge F_n y_n x\}\,,$$

the predicate in question can be written as '$\langle F, G\rangle{:}H$', making use of the general definition of the Pierce product:

$$\{x_1 \ldots x_m \mid \exists y_1 \ldots y_n(Fy_1 \ldots y_n x_1 \ldots x_m \wedge Gy_1 \ldots y_n)\}\,.$$

[1] Beware! This definition of functionality follows Peano, Gödel, Tarski, and Quine contrary to the nowadays widespread convention to call 'F' functional iff $\forall xyz(Fxy \wedge Fxz \rightarrow y = z)$. Our definition of the Pierce product is a consequence of this decision.

The third type of predicate is subsumed by the second, with 'H' replaced by the identity predicate 'id', i.e. '$\{xy \mid x = y\}$'. A better known notation for '$\langle F, G\rangle$:id' is '$F \doteq G$'.

Reflexivity of attributes can be expressed using identity as an attribute because '$\langle F, \mathrm{id}\rangle$:id' is equivalent to '$\{x \mid Fxx\}$'. Finally, *attribute composition* is reverse relational composition. Notation: '$F|G$' for '$G \circ F$', i.e. for '$\{xy \mid \exists z(Fzy \wedge Gxz)\}$'.

Attribute-Value Logic

Attribute-value logic can be regarded as a *predicate-operator logic* in the sense of Quine (e.g. 1976). The following schemata are valid without additional assumptions as one easily verifies:[2]

(1)
a. $P|\mathrm{id} \;\equiv\; \mathrm{id}|P \;\equiv\; P$
b. $(P|Q)|R \;\equiv\; P|(Q|R)$
c. $P{:}(Q{:}A) \;\equiv\; (P|Q){:}A$
d. $P{:}(A \vee B) \;\equiv\; P{:}A \vee P{:}B$
e. $\mathrm{id}{:}A \;\equiv\; A$
f. $P{:}\Lambda \;\equiv\; \Lambda$
g. $P{:}\mathrm{V} \;\equiv\; \langle P, P\rangle{:}\mathrm{id}$

Here, '$F \equiv G$' stands for '$\forall x(Fx \leftrightarrow Gx)$', '$F \vee G$' for '$\{x \mid Fx \vee Gx\}$', etc., as well as 'V' and 'Λ' for '$\{x \mid x = x\}$' and '$\{x \mid x \neq x\}$' respectively.

Using rules of inference like disjunction and transitivity and the fact that $A \subseteq B$ iff $B \equiv A \vee B$, a consequence of (1d) is that

(2) $$\text{if } A \subseteq B \text{ then } P{:}A \subseteq P{:}B\,.$$

To put predicate-operator logic to work once more, apply (2) to the valid schema '$Q{:}\mathrm{V} \subseteq \mathrm{V}$', which leads to (3c) using (1g) since, with (1c) and (1g), $\langle P|Q, P|Q\rangle{:}\mathrm{id} \equiv P{:}(Q{:}\mathrm{V})$. A further simple example is the derivation of (3a) from (1e) and (1g).

(3)
a. $\mathrm{V} \;\subseteq\; \langle \mathrm{id}, \mathrm{id}\rangle{:}\mathrm{id}$
b. $\langle P, Q\rangle{:}\mathrm{id} \;\subseteq\; \langle Q, P\rangle{:}\mathrm{id}$
c. $\langle P|Q, P|Q\rangle{:}\mathrm{id} \;\subseteq\; \langle P, P\rangle{:}\mathrm{id}$
d. $\langle P_1, \ldots, P_n\rangle{:}A \;\subseteq\; \langle P_i, P_i\rangle{:}\mathrm{id}$

Presupposing attribute predicates as functional implies additional valid schemata, with proofs again an easy exercise.

(4)
a. $\langle P, Q\rangle{:}\mathrm{id} \wedge \langle Q, R\rangle{:}\mathrm{id} \;\subseteq\; \langle P, R\rangle{:}\mathrm{id}$
b. $\langle P, Q\rangle{:}\mathrm{id} \wedge \langle P|R, P|R\rangle{:}\mathrm{id} \;\subseteq\; \langle P|R, Q|R\rangle{:}\mathrm{id}$
c. $\langle P_1, Q_1\rangle{:}\mathrm{id} \wedge \ldots \wedge \langle P_n, Q_n\rangle{:}\mathrm{id} \wedge \langle P_1, \ldots, P_n\rangle{:}A \;\subseteq\; \langle Q_1, \ldots, Q_n\rangle{:}A$

(5)
a. $P{:}(A \wedge B) \;\equiv\; P{:}A \wedge P{:}B$
b. $P|\langle Q_1, \ldots, Q_n\rangle \;\equiv\; \langle P|Q_1, \ldots, P|Q_n\rangle$
c. $P{:}(\langle Q_1, \ldots, Q_n\rangle{:}A) \;\equiv\; \langle P|Q_1, \ldots, P|Q_n\rangle{:}A$

Attribute-Value Theories

If not otherwise specified, assume a fixed attribute-value signature, that is, a set L of elementary attribute predicates and sets A_n of n-adic sort predicates. Let in addition 'V' and 'Λ' count as monadic sort predicates and 'id' both as elementary attribute and as dyadic sort predicate. 'V', 'Λ', and 'id' will henceforth be denoted respectively by '$\top$', '$\perp$', and '$\mathbb{I}$'.

Primitive attribute-value descriptions are of the form $\ulcorner\langle p_1, \ldots, p_n\rangle{:}a\urcorner$, where the p_i's are possibly composed attribute predicates and a is an n-adic sort predicate. Attribute-value descriptions are then inductively constructed from these primitives by conjunction, disjunction, and attribute prefixing.

Attribute-value axioms are universally quantified conditionals, whose antecedents and consequents are attribute-value descriptions. An *attribute-value theory*

[2]Schemata (1a) to (1e) are part of the definition of *Boolean modules* and *Pierce algebras*, (1f) ensures normality; see e.g. Brink et. al., 1997.

is a set of such axioms. Since AV axioms are of the form $\ulcorner \phi \subseteq \psi \urcorner$, with ϕ and ψ AV descriptions, they can be regarded as ordered pairs of AV descriptions. AV theories thus correspond to dyadic relations on the set of AV descriptions. Notation: $\phi \preccurlyeq_T \psi$ iff $\ulcorner \phi \subseteq \psi \urcorner \in T$. An AV theory is *closed with respect to attribute prefixing* iff $p{:}\phi \preccurlyeq p{:}\psi$ whenever $\phi \preccurlyeq \psi$.[3]

Head-Driven Phrase Structure Grammar (HPSG) serves as an example. Typical axioms are: *sign* $\preccurlyeq$ *word* $\vee$ *phrase*, *word* $\wedge$ *phrase* $\preccurlyeq \bot$, and:

$$\text{DTRS}{:}\,\textit{headed-struc} \preccurlyeq \\ \langle \text{SYNSEM}|\text{LOC}|\text{CAT}|\text{HEAD}, \text{DTRS}|\text{HEAD-DTR}|\text{SYNSEM}|\text{LOC}|\text{CAT}|\text{HEAD}\rangle{:}\mathbb{I}\,.$$

An AV theory of particular importance is the theory *Avl* given by the following axiom schemata.

$$\begin{array}{lr}
\top \preccurlyeq \langle \mathbb{I}, \mathbb{I}\rangle{:}\mathbb{I} & (\textit{trivial reflexivity}) \\
\langle p, q\rangle{:}\mathbb{I} \preccurlyeq \langle q, p\rangle{:}\mathbb{I} & (\textit{symmetry}) \\
\langle p, q\rangle{:}\mathbb{I} \wedge \langle q, r\rangle{:}\mathbb{I} \preccurlyeq \langle p, r\rangle{:}\mathbb{I} & (\textit{transitivity}) \\
\langle p, q\rangle{:}\mathbb{I} \wedge \langle p|r, p|r\rangle{:}\mathbb{I} \preccurlyeq \langle p|r, q|r\rangle{:}\mathbb{I} & (\textit{substitutivity}) \\
\langle p|q, p|q\rangle{:}\mathbb{I} \preccurlyeq \langle p, p\rangle{:}\mathbb{I} & (\textit{prefix closure}) \\
\langle p_1, \ldots, p_n\rangle{:}a \preccurlyeq \langle p_1, p_1\rangle{:}\mathbb{I} \wedge \ldots \wedge \langle p_n, p_n\rangle{:}\mathbb{I} & (\textit{reflexivity}) \\
\langle p_1, q_1\rangle{:}\mathbb{I} \wedge \ldots \wedge \langle p_n, q_n\rangle{:}\mathbb{I} \wedge \langle p_1, \ldots, p_n\rangle{:}a \preccurlyeq \langle q_1, \ldots, q_n\rangle{:}a & (\textit{substitutivity})
\end{array}$$

Since *Avl* collects (3) and (4) above, its axioms are theorems of the first-order theory *Fun* given by the axiom schema of attribute functionality. *Avl* will turn out to be complete with respect to first-order derivability in *Fun*.

Attribute-Value Systems

Since attribute-value theories are first-order theories, there is a standard model theoretic approach towards their semantics. Simply add *Fun*, which serves as background assumption, and take models of the extended theory.

If T is an AV theory then first-order models of $T \cup \mathit{Fun}$ are called *attribute-value systems of T*. Such a model consists of a set U (the "universe") and a function M taking elements of L and A_n respectively to functional relations and n-adic relations on U. M extends as usual to first-order formulae and thus commutes with operators. For example, $M(p{:}\phi) = M(p){:}M(\phi)$.[4] Furthermore it is required that if $\phi \preccurlyeq_T \psi$ then $M(\phi) \subseteq M(\psi)$. Notice that every AV system is one of *Avl* because *Avl* axioms are *Fun* theorems, and that an AV system of a theory T is also one of the prefixing closure of T in view of (2) above.

Algebraizing Attribute-Value Theories

Identifying descriptions that are equivalent with respect to a theory T leads to the *Lindenbaum-Tarski algebra* of T. The standard quotient construction takes the set of equivalence classes and proves the induced algebraic operations to be well-defined. For our purposes the following step-by-step construction is more convenient.

Attribute composition is "algebraized" by regarding classes of equivalent attributes as elements of the free monoid L^* over L with operation $|$ and unit $\mathbb{I}$. The set of primitive AV descriptions can then be specified as (a representation of) $L^* \times L^* \uplus \biguplus_{n>0} (L^*)^n \times A_n$, with operations $\langle\ \rangle$, and $:$ restricted appropriately. Here we make use of the observation that $\langle p, q\rangle{:}\mathbb{I}$ can be seen as a representation of the ordered pair of p and q, which gives rise to the first direct summand.

[3] Closure with respect to prefixing virtually resembles the so-called *master modality* applied to conditional constraints; see e.g. Rounds, 1997.

[4] It is important to note that the expression to the right of '=' denotes the Pierce product of *sets*. In other words, M interpretes predicate abstracts as names of subsets of U.

General AV descriptions are now obtained as elements of the distributive lattice freely generated by the set of primitives. Prefixing by p corresponds to the lattice homomorphism induced by the function defined for generators as follows: $p{:}(\langle q_1,\ldots,q_n\rangle{:}a) = \langle p|q_1,\ldots,p|q_n\rangle{:}a$. It remains to add $\top$ and $\bot$.

So far, identification is limited to conjunction, disjunction, and prefixing. Neither *Avl* nor any other AV theory is taken into account. This will be done below within the more general setting of possibly *infinite* disjunctions. Though not employed here it would allow, for example, to include regular attribute equations. And from a technical perspective the gain clearly makes up for the load.

Geometric Theories and Classifying Locales

According to Vickers (1999), *propositional geometric formulae* over a set G of primitives or *subbasics* are built from G by $\wedge$ and $\bigvee$ including $\top$ and $\bot$. More exactly, formulae are elements of the *frame* $\mathcal{F}(G)$ *freely generated by* G. In this context, a *frame* is a poset, ordered by $\leqslant$, with finite meets and arbitrary joins such that $a \wedge \bigvee S = \bigvee\{a \wedge b \mid b \in S\}$. *Frame homomorphisms* are functions preserving finite meets and arbitrary joins. $\mathcal{F}(G)$ is characterized up to isomorphism by the property that there is a function η from G to $\mathcal{F}(G)$ such that every function v from G to a frame A factors uniquely through η and a frame homomorphism $\overline{v}$ from $\mathcal{F}(G)$ to A. $\mathcal{F}(G)$ is, for example, the ideal completion of the distributive lattice freely generated by G.[5]

A *propositional geometric theory* T consists of a set G and a dyadic relation $\preccurlyeq$ on $\mathcal{F}(G)$. Members of $\preccurlyeq$ are *axioms* of T. The theory is *finitary* if the formulae of its axioms are finitely constructed from G by meet and join. A *model of* T *in a frame* A or A*-valued model of* T is a function v from G to A such that if $a \preccurlyeq b$ then $\overline{v}(a) \leqslant \overline{v}(b)$. A model of T in a frame is *universal* iff every model of T factors uniquely through it by a frame homomorphism. Such a frame $\mathcal{U}(T)$ is called *the frame presented by* T. It can be constructed by *geometric deduction* as the quotient frame of $\mathcal{F}(G)$ modulo the closure of $\preccurlyeq$ with respect to *reflexivity* and *transitivity* such that $a \preccurlyeq b \wedge c$ iff $a \preccurlyeq b$ and $a \preccurlyeq c$, and $\bigvee S \preccurlyeq b$ iff $a \preccurlyeq b$ for every element a of S. If T is finitary, one can proceed first without infinite joins and then do ideal completion.

A subset of G is T*-saturated* if its characteristic function is a $\mathbf{2}$-valued model of T, with $\mathbf{2} = \{\bot, \top\}$. Let $A^\star$ be the set of frame homomorphisms from a frame A to $\mathbf{2}$. By definition, there is a one-to-one correspondence between $\mathcal{U}(T)^\star$ and the set of $\mathbf{2}$-valued models of T.

Following Vickers (1989) we define a locale to be a topological system of a certain form. A *topological system* consists of a set X of *points*, a frame A of *opens*, and a *satisfaction* relation $\vDash$ borne by points to opens respecting meet and join. A *locale* is a topological system isomorphic to one with frame A, point space $A^\star$, and satisfaction defined such that $x \vDash a$ iff $x(a) = \top$.

The *classifying locale* $\mathcal{L}(T)$ of a theory T is a/the locale whose frame is presented by T. Its points thus are the $\mathbf{2}$-valued models of T, the T*-saturated* subsets of G, or any other equivalent representation with satisfaction defined appropriately.

Theorem: The classifying locale of a finitary theory *has enough points*, that is, if b is satisfied by every point satisfying a then $a \leqslant b$ ∎[6]

In other words, finitary geometric deduction is *complete*.

[5]e.g. Johnstone, 1982.

[6]See e.g. Johnstone (1982) or Vickers (1989) for a proof, which employs, unsurprisingly, the Prime Ideal Theorem.

Attribute-Value Structures and Completeness

An attribute-value theory T can be seen as a geometric theory over the set of primitive attribute-value descriptions. Points of the classifying locale $\mathcal{L}(Avl \cup T)$ are called (*abstract*) *attribute-value structures of* T.

Each AV structure x determines an AV system, i.e. a first-order model of *Fun* as follows.[7] First note that the set of subbasics satisfied by x is Avl-saturated and that its intersection with $L^* \times L^*$ is a conditional right congruence with respect to $|$ with prefix-closed field. Now take $\{[p] \mid x \vDash \langle p, p\rangle : \mathbb{I}\}$, where $[p]$ is the congruence class of p, as universe U_x, and define M_x such that

$$\begin{array}{rcl} M_x(l) & = & \{([p|l],[p]) \mid x \vDash \langle p|l, p|l\rangle : \mathbb{I}\} \\ M_x(a) & = & \{([p_1],\ldots,[p_n]) \mid x \vDash \langle p_1,\ldots,p_n\rangle : a\} \end{array}$$

for elementary attributes l and n-adic sort predicates a. One easily checks that $M_x(l)$ and $M_x(a)$ are well-defined and that $M_x(l)$ is functional.

Lemma 1: $[\mathbb{I}] \in M_x(\langle p_1,\ldots,p_n\rangle : \phi)$ iff $([p_1],\ldots,[p_n]) \in M_x(\phi)$.

Proof: $M_x(\langle p_1,\ldots,p_n\rangle : \phi)$ coincides with $\langle M_x(p_1),\ldots,M_x(p_n)\rangle : M_x(\phi)$, and $[\mathbb{I}]$ is an element of the latter iff, by definition, there are congruence classes $u_1,\ldots,u_n$ such that $(u_i,[\mathbb{I}]) \in M_x(p_i)$ and $\langle u_1,\ldots,u_n\rangle \in M_x(\phi)$, in which case u_i necessarily equals $[p_i]$ since $M_x(p_i)$ is functional ∎

"Truth Lemma": $[\mathbb{I}] \in M_x(\phi)$ iff $x \vDash \phi$.

Proof: According to Lemma 1, $[\mathbb{I}] \in M_x(\langle p_1,\ldots,p_n\rangle : a)$ iff $([p_1],\ldots,[p_n]) \in M_x(a)$, that is, by definition, iff $x \vDash \langle p_1,\ldots,p_n\rangle : a$. Furthermore, M_x and satisfaction both respect meet and join ∎

Lemma 2: If T is a prefixing-closed AV theory then for every AV structure x of T, U_x and M_x define an AV system of T.

Proof: Suppose that $\phi \preccurlyeq_T \psi$ and $[p] \in U_x$. By assumption, if $x \vDash p{:}\phi$ then $x \vDash p{:}\psi$. By Truth Lemma and Lemma 1 this implies that if $[p] \in M_x(\phi)$ then $[p] \in M_x(\psi)$ ∎

The following *completeness* theorem recaptures and slightly generalizes Moshier (1993) and Osswald (1999).

Theorem: If T is a finitary attribute-value theory then finitary geometric derivability in $T \cup Avl$ plus prefixing is equivalent to first-order derivability in $T \cup Fun$.

Proof: It remains to prove completeness of geometric derivability plus prefixing. Suppose $T \cup Fun \vdash \phi \subseteq \psi$. For every AV structure x of the prefixing-closure $\overline{T}$ of T, M_x determines according to Lemma 2 a model of $T \cup Fun$. By Truth Lemma, if $x \vDash \phi$ then $[\mathbb{I}] \in M_x(\phi)$, and therefore $[\mathbb{I}] \in M_x(\psi)$, that is, $x \vDash \psi$. Thus $\phi \leqslant \psi$ in $\mathcal{U}(\overline{T} \cup Avl)$ because the classifying locale of a finitary theory has enough points ∎

Negation and Implication

There are several possibilities to express *negation* within geometric logic, for example as axioms $a \preccurlyeq \bot$. Or introduce additional primitives $-a$ plus axiom schemata $a \wedge -a \preccurlyeq \bot$ and $\top \preccurlyeq a \vee -a$.

As for a *conditional*, note that frames are, as ordered sets, indistinguishable from complete Heyting algebras, with $a \Rightarrow b = \bigvee\{c \mid c \wedge a \leqslant b\}$. But beware: $\top \preccurlyeq a \Rightarrow b$ has in general not the same effects as $a \preccurlyeq b$. Overlooking this difference has led Pollard and Sag (1987) to an inadequate conception of conditional information; see Osswald (1999).

[7] The construction if of course standard; cf. e.g. Rounds, 1997.

Reflections

One might ask what we have revealed about the semantics of attribute-value theories since we neither did consider the meaning of specific sort and attribute predicates, nor the nature of the entities those predicates are ascribed to, nor criteria for justifying such ascriptions. These are of course central issues for an empirical theory about a certain domain as e.g. HPSG intends to be one for linguistics. What we did, at least, is to straighten the discussion of such questions by explicating the referential structure of AV descriptions as regimented and formalized natural language predicates. Moreover, in our formulation AV theories consist of universally quantified conditionals and thus are of a form desirable for scientific theories.

Our main point was to define the formal denotation of an AV theory, or, to put it another way, to reveal *the ontology an AV theory defines by itself*. As emphasized by Quine (e.g. 1969), a theory determines an identity predicate by *identification of indiscernibles* and tying down criteria of identity is all what matters to ontology. For an AV theory T this means to look for a set X of "generic" entities together with a satisfaction relation borne by these entities to AV descriptions such that two elements x and y of X are identical iff they satisfy the same descriptions modulo equivalence in $T \cup Fun$. A further requirement which suggests itself is to choose X "as large as possible" so that for every pair of non-equivalent AV descriptions there is an element of X satisfying one of them but not the other. On account of our completeness result these two properties determine X to be the point space of a classifying locale of (the prefixing-closure of) $T \cup Avl$, whose uniqueness up to isomorphism nicely reflects Quine's conception of *ontological indifference*.

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